Numerical Study of Unsteady MHD Pulsatile Flow through Porous Medium in an Artery Using Generalized Differential Quadrature Method (GDQM)

I. M. Eldesoky, M. H. Kamel, Reda M. Hussien, and Ramzy M. Abumandour

***Abstract*—The unsteady pulsatile flow of blood through porous medium has been studied under the influence of periodic body acceleration by considering blood as incompressible Newtonian electrically conducting fluid in the presence of magnetic field. A numerical solution of the equation of motion is obtained by applying a generalized differential quadrature method (GDQM), to derivatives with respect to space variables of differential equations and for the time derivative applying 4th order Runge Kutta Method. This combination of DQM and 4th order RK method gives very good numerical technique for solving time dependent problems. The algorithm is coded in Matlab 7.14.0.739 and the simulations are run on a Pentium 4 CPU 900 MHz with 1 GB memory capacity. The numerical results show and discussed with the help graphs. The study show that the axial velocity of the blood increases with increasing the permeability parameter of porous medium and the Womersley parameter, and decreases with increasing the Hartmann number. The study is useful for evaluating the role of porosity when the body is subjected to magnetic resonance imaging (MRI).**

***Index Terms*—Pulsatile blood flow magnetic field, body acceleration, porous medium, differential quadrature method, runge-kutta method.**

I. Introduction

MHD viscous flow though pipes plays significant role in different areas of science and technology such as Petroleum industry, Biomechanics, Drainage and Irrigation engineering and so on. The investigations of blood flow through arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis. The pulsatile flow of blood through an artery has drawn the attention to the researchers for a long time due to its great importance in medical sciences. Under normal conditions, blood flow in the human circulatory system depends upon the pumping action of the heart and this produces a pressure gradient throughout the arterial network [1], [2]. During the last decades extensive research work has been done on the fluid dynamics of biological fluids in the presence of magnetic field. The flow of a conducting fluid in a circular pipe has been investigated by many authors [3]–[5]. References [6], [7] have studied steady viscous incompressible flow through circular and elliptic tubes under the influence of periodic pressure. They studied the effect of MHD flow of blood under body acceleration. Also, studied Womersley problem for pulsatile flow of blood through porous medium.

Many researchers have reported that the blood is an electrically conducting fluid [8], [9]. The electromagnetic force (Lorentz force) acts on the blood and this force opposes the motion of blood and there by flow of blood is impeded, so that the external magnetic field can be used in the treatment of some kinds of diseases like cardiovascular diseases and in the diseases with accelerated blood circulation such as hemorrhages and hypertension.

In general, biological systems are affected by an application of external magnetic field on blood flow through human arterial system. Many mathematical models have already been investigated by several research workers to explore the nature of blood flow under the influence of an external magnetic field. Reference [10] studied a mathematical model of biomagnetic fluid dynamics (BFD), suitable for the description of the Newtonian blood flow under the action of magnetic field. Reference [11] studied magneto-hydrodynamic effects on blood flow through a porous channel. They considered the blood a Newtonian fluid and conducting fluid. Arterial MHD pulsatile flow of blood under periodic body acceleration has been studied by [12]. The effect of uniform transverse magnetic field on its pulsatile motion through an axi-symmetric tube is analyzed by [13]. Reference [14] studied the characteristics of blood flow under body accelerations. References [15]–[17] considered various types of body accelerations and studied different characteristics of blood flow according to the nature of accelerations. Reference [18] discussed the flow characteristics of blood under external body acceleration assuming blood to be a Newtonian fluid. References [20], [21] studied the effect of body acceleration in different situations.

Numerical approximation methods for solving partial differential equations have been widely used in various engineering fields. Classical techniques such as finite element and finite difference methods are well developed and well known. These methods can provide very accurate results by using a large number of grid points. In seeking an alternate numerical method using fewer grid points to find results with acceptable accuracy, the method of DQM was introduced by [26]–[35].

In this paper, DQM and 4th order RKM is applied to time dependent problem. DQ technique approximates the derivative of a function at a grid point by a linear weighted summation of all the functional values. Derivatives with

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respect to space variables are discretized using DQM giving a system of ordinary differential equations for time derivative, and time derivatives are discretized using 4th order RKM. This combination of DQM and 4th order RK method gives very good numerical technique for solving time dependent problems. Stability of 4th order RKM criterias are controlled with several values of time increment *Δt* and number of grid points *N* in space region. In this paper, GDQM used for studying the unsteady pulsatile blood flow through porous medium under the influence of periodic body acceleration in the presence of magnetic field.

II. Mathematical Formulation

Consider the flow in an artery of radius *R* as shown in Fig. 1, here blood is supposed to be as an electrically conducting, Newtonian, incompressible, and viscous fluid in the presence of magnetic field acting along the radius of a circular pipe. Also, the viscosity of blood is considered to be constant. We assume that the magnetic Reynolds number of the flow is taken to be small enough, so that the induced magnetic and electric field can be neglected. We consider the flow as axially symmetric, pulsatile and fully developed as [22] in dimensionless form.

where A_O and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_O is the amplitude of the body acceleration, $\omega_p = 2\pi f_p$, $\omega_b = 2\pi f_b$ with f_p is the pulse frequency, and f_b is the body acceleration frequency and t is time, $u(r,t)$ is the velocity distribution, ρ the blood density, μ the dynamic viscosity of the blood, $\overline{B} = (0, B_O, 0)$ the magnetic field, k is the permeability parameter of porous medium, and σ the electric conductivity of the blood.

The Hartmann number, *Ha* and the Womersley parameter, *α* are defined respectively by:

$$H_a = B_O R \sqrt{\frac{\sigma}{\mu}},$$

$$\alpha = R\sqrt{\frac{\rho\omega}{\mu}}. \tag{1}$$

$$\alpha^2 \frac{\partial u}{\partial t} = A_O + A_1 \cos(t) + a_O \cos(bt) + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \left(H_a{}^2 + \frac{1}{k}\right)u. \tag{2}$$

Also the boundary conditions are:

$$u(1,t) = 0 \quad \text{on} \quad r = 1, \quad (\text{no slip}) \tag{3a}$$

$$u(0,t) \quad \text{is finite} \quad (\text{axis of the pipe}) \tag{3b}$$

And the initial condition is:

$$u(r,0) = 1 \quad \text{at} \quad t = 0 \tag{3c}$$

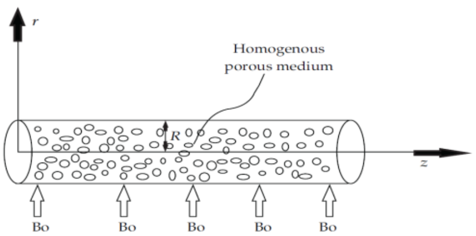

Fig. 1. Schematic diagram for the flow geometry.

III. Generalized Differential Quadrature Method (GDQM)

The DQM is a numerical solution technique for initial and/or boundary value problems. This technique has been successfully employed in a variety of problems in engineering and physical sciences. The DQM approximates the derivative of a function at any location by a linear summation of all the functional values along a mesh (grid) line. The GDQM is systematically employed to solve problems in Fluid mechanics, vibration analysis and structural analysis. The technique of GDQM for the solution of partial differential equations extended and generalized. Numerical examples have shown the super accuracy, efficiency, convenience and the great potential of this method. A GDQM, which was recently proposed by [28]–[30] for solving partial differential equations. For the discretization of the first and higher order derivatives, the following linear constrained relationships are applied

$$f_x^{(n)}(x_i, t) = \sum_{j=1}^{N} C_{ij}^{(n)} . f(x_j, t), \quad n = 1, 2, ..., N-1, \tag{4}$$

for $i = 1, 2, ..., N$; where $f_x^{(n)}$ indicate n^{th} order derivatives of $f(x,t)$ with respect to x at x_i, N is the number of grid points in the whole dominant $C_{ij}^{(n)}$ are the weighting coefficients. The key to DQ is to determine the weighting coefficients for the discretization of a derivative of any order. In order to find a simple algebraic expression for calculating the weighting coefficients without restricting the choice of grid meshes, [28]–[30] gave a convenient and recurrent formula for determining the derivative weighting coefficients.

To determine the weighting coefficients of the GDQ method as:

Weighting coefficients for the first order derivative

$$C_{ij}^{(1)} = \frac{M_N^{(1)}(x_i)}{(x_i - x_j).M_N^{(1)}(x_j)}, \quad i, j = 1, 2, \cdots, N \text{ and } i \neq j \tag{5}$$

$$C_{ii}^{(1)} = -\sum_{j=1, j\neq i}^{N} C_{ij}^{(1)}, \quad i = 1, 2, \cdots, N \tag{6}$$

where

$$M(x) = (x - x_1)(x - x_2)\cdots(x - x_N) \tag{7a}$$

$$M^{(1)}(x_i) = \prod_{k=1, k\neq i}^{N} (x_i - x_k) \tag{7b}$$

Weighting coefficients for the second and higher order derivatives

$$C_{ij}^{(n)} = n\left(C_{ij}^{(1)}.C_{ii}^{(n-1)} - \frac{C_{ij}^{(n-1)}}{x_i - x_j}\right),$$

$$\text{for } j \neq i, \quad i, j = 1, 2, ..., N; \quad n = 2, 3, ..., N-1. \tag{8}$$

$$C_{ii}^{(n)} = -\sum_{j=1, j\neq i}^{N} C_{ij}^{(n)}, \text{ for } i, j = 1, 2, ..., N; \quad n = 2, 3, ..., N-1. \tag{9}$$

where $C_{ij}^{(n)}$ and $C_{ij}^{(n-1)}$ are the weighting coefficients of the n^{th} and the $(n-1)^{th}$ derivatives. Thus (8) and (9) together with (5) and (6) give a convenient and general form for determining the weighting coefficients for the derivatives of orders one through *N–1*.

IV. NUMERICAL DISCRETIZATION AND STABILITY OF THE SCHEME

In the present study, substituting the DQ derivative approximations given in (4) in the governing (2). The coordinates of the grid points are chosen according to Chebyshev-Gauss-Lobatto by using *N* sampling as:

$$X(i) = \frac{1}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right], \qquad i = 1,2,3,...,N;$$

The GDQM is applied for the discretization of space derivatives of the unknown function *u*, we obtain the ordinary differential equation

$$\alpha^2 \frac{\partial u(r_i,t)}{\partial t} = A_O + A_1\cos(t) + a_O\cos(bt) + \sum_{j=1}^{N} C_{i,j}^{(2)} \cdot u_j + \frac{1}{r_i}\sum_{j=1}^{N} C_{i,j}^{(1)} \cdot u_j - \left(H_a{}^2 + \frac{1}{k}\right)u_i, \; i = 1,2,\ldots,N \tag{10}$$

where u_i, $i = 1,2,...,N$; is the velocity value at the grid r_i, $C_{ij}^{(1)}$ and $C_{ij}^{(2)}$ are the weighting coefficient matrix of the first and second order derivatives. Similarly, the derivatives in the boundary conditions can be discretized by the GDQM. As a result, the numerical boundary conditions can be written as:

$$\sum_{k=1}^{N} C_{1,k}^{(1)} \cdot u_k = 0 \tag{11a}$$

$$u_N = 0 \tag{11b}$$

Equation (11b) can be easily substituted into the governing equation. This is not the case for (11a). However, one can give solutions, u_1 as

$$u_1 = -\frac{1}{C_{11}^{(1)}} \cdot \sum_{k=2}^{N-1} C_{1,k}^{(1)} \cdot u_k \tag{12}$$

According to (12), u_1 is expressed in terms of $u_2, u_3, \cdots, u_{N-1}$, and can be easily substituted into the governing equation. It should be noted that (11) provides two boundary equations. In total we have *N* unknowns $u_1, u_2, \cdots, u_N$. In order to close the system, the discretized governing (10) has to be applied at $N-2$ mesh points. This can be done by applying (10) at grid points $r_2, r_3, \cdots, r_{N-1}$. Substituting (11b) and (12) into (10) gives:

$$\alpha^2 \frac{\partial u(r_i,t)}{\partial t} = A_O + A_1\cos(t) + a_O\cos(bt) + \sum_{j=2}^{N-1} C_{i,j}^{(2)} \cdot u_j + \frac{1}{r_i}\sum_{j=2}^{N-1} C_{i,j}^{(1)} \cdot u_j - \left(H_a{}^2 + \frac{1}{k}\right)u_i, \; i = 2,3,\ldots,N-1 \tag{13}$$

It is noted that (13) has $N-2$ equations with $N-2$ unknowns.

Now, the discretization for time derivative will be performed by using Runge-Kutta Method. Now, $\frac{\partial u}{\partial t}$ is also considered discretized as $\frac{\partial u_{ij}}{\partial t}$, thus (13) is a set of DQ algebraic equations which can be written in a matrix form

$$[A]\{u\} = \{b\}, \tag{14}$$

where $\{u\} = \{u_2, u_3, \cdots, u_{N-1}\}$ is a vector of unknown $N-2$ functional values at all discretized points of the region, $[A]$ is the $(N-2)\times(N-2)$ coefficient matrix, and the right hand side vector $\{b\}$ of size $(N-2)\times 1$ contains first order time derivatives of the function *u* at the same discretized points. Therefore a numerical scheme is necessary for handling these time derivatives. (14) can be solved by several time integration schemes such as Euler, Modified Euler, and Runge-Kutta Methods. Here, Runge-Kutta Method is going to be used since it is a one step method obtained from the Taylor series expansion of u up to and including the terms involving $(\Delta t)^4$ where Δt is the step size with respect to time. The 4th order RKM since its stability region is larger comparing to the other time integration methods and simple for the computations.

The resulting algebraic system of (14) can originally be considered as an initial value problem in the form (a set of ordinary differential equations in time)

$$\{b\} = \frac{d\{u\}}{dt} = [A]\{u\} \tag{15}$$

Thus the 4th order RKM gives for the governing equation the following vector equation

$$\{u_{n+1}\} = \{u_n\} + \frac{\Delta t}{6}\left[\{K_1\} + 2\{K_2\} + 2\{K_3\} + \{K_4\}\right] \tag{16}$$

where,

$$K_1 = f(t_n, u_n),$$

$$K_2 = f\left(t_n + \frac{\Delta t}{2}, u_n + \frac{\Delta t}{2}K_1\right),$$

$$K_3 = f\left(t_n + \frac{\Delta t}{2}, u_n + \frac{\Delta t}{2}K_2\right),$$

$$K_4 = f(t_n + \Delta t, u_n + \Delta t K_3)$$

Applying 4th order RKM (16) in (15). Thus, we can easily write by taking $[A]\{u\}$ as the vector function $\{f(t,\{u\})\}$ in the sample initial value problem $\dot{u} = f(t,u)$ So,

$$\{f(t,\{u\})\} = [A]\{u\} \tag{17}$$

The Matlab program has been used to solve this problem and get the velocity distribution.

V. NUMERICAL RESULTS AND DISCUSSION

We studied unsteady pulsatile blood flow through porous medium in an artery under the influence of periodic body acceleration in the presence of magnetic field. We have shown the relation between the different parameters of motion such as Hartmann number *Ha*, Womersley parameter *α*, the frequency of body acceleration *b*, the permeability parameter of porous medium k with the velocity distribution to investigate the effect of changing these parameters on the blood flow. Hence, we can be controlling the process of flow.

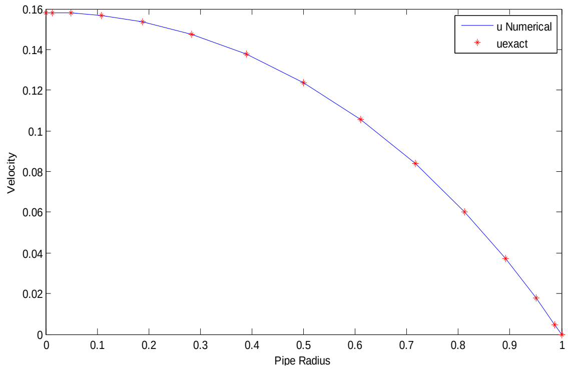

Fig. 2. Comparisons between exact and numerical solution of velocity with the pipe radius [A_O=1, A_1=0, a_O=0, b=0, Ha=1, $1/k$=2, $t\to\infty$].

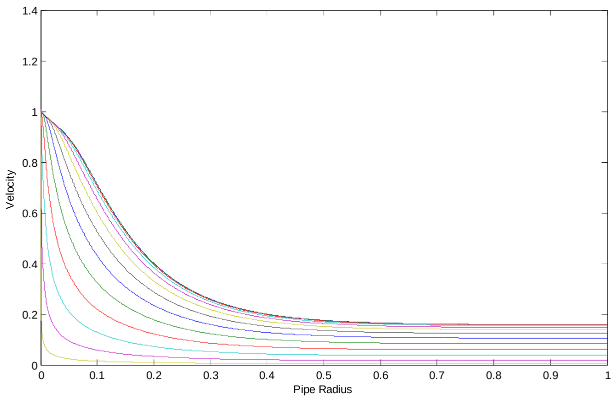

Fig. 3. Comparisons between exact and Numerical Solution of velocity distribution with time [α=1, A_O=1, A_1=0, a_O=0, b=0, Ha=1, $1/k$=2].

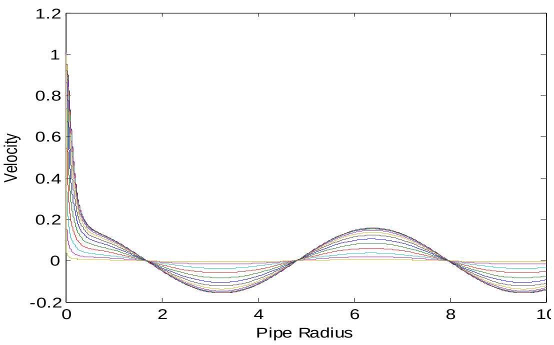

Fig. 4. The Velocity Distribution with Time [α=1, A_O=1, A_1=1, a_O=0, b=0, Ha=1, $1/k$=2].

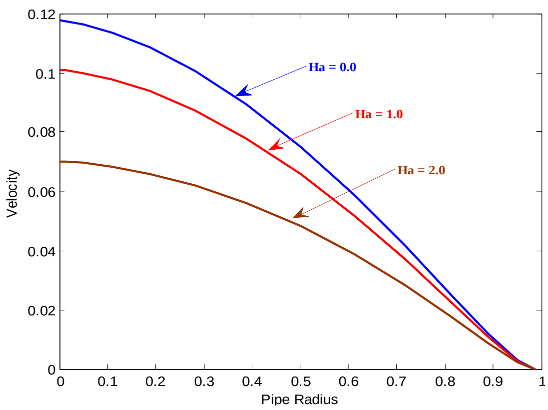

Fig. 5. The effect of Hartman number on velocity distribution [α=1, A_O=1, A_1=1, a_O=0, b=0, $1/k$=2, t=1].

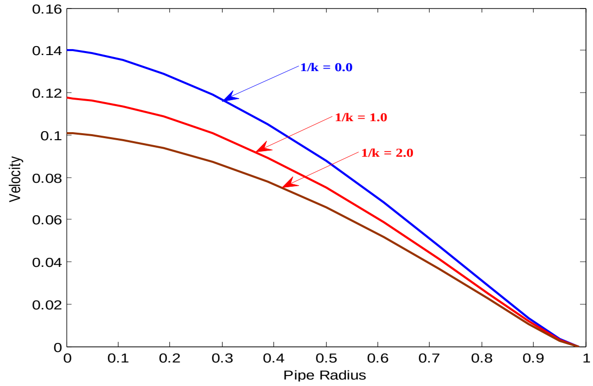

Fig. 6. The effect of Porosity number on velocity distribution [α=1, A_O=1, A_1=1, a_O=0, b=0, Ha=1, t=1].

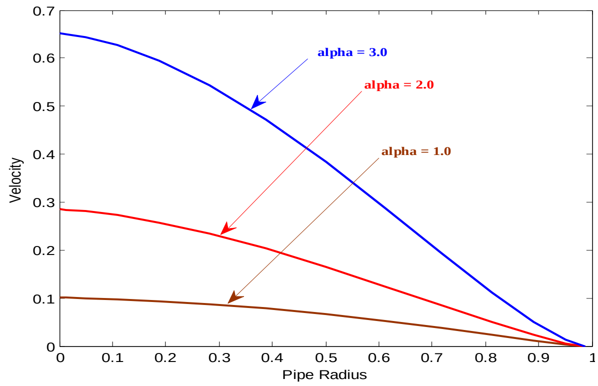

Fig. 7. The effect of Womersley parameter on velocity distribution [A_O=1, A_1=1, a_O=0, b=0, Ha=2, $1/k$=1, t=1].

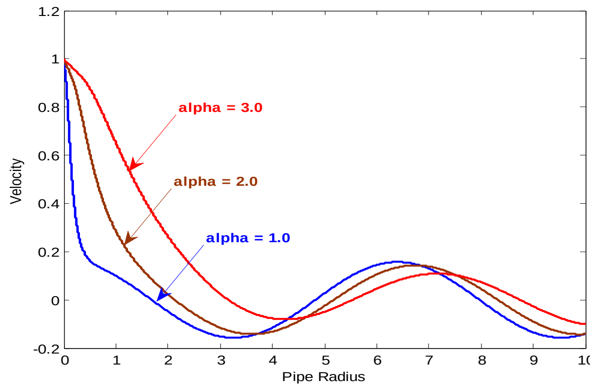

Fig. 8. The effect of Womersley parameter on velocity distribution with time [A_O=1, A_1=1, a_O=0, b=0, Ha=1, $1/k$=2, t=1].

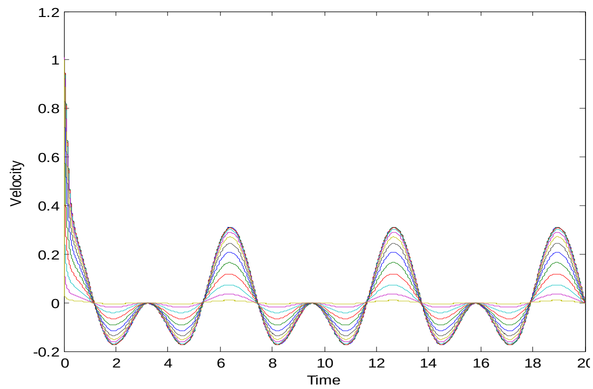

Fig. 9. The Velocity Distribution with Time [α=1, A_O=1, A_1=1, a_O=1, b=2, Ha=1, $1/k$=2].

A numerical code has been written to calculate the velocity distribution according to (14). In order to check our code and comparison the numerical solution with exact solution, we run it for the parameters related to a realistic physical problem similar to the one used by [24], [25], Figs. 2, 3 for the motion of a conducting fluid through a porous medium, Fig. 2 shows the relation between velocity distribution with the pipe radius, for $r=0.5$ we obtain the axial velocity $u=0.12374654323755$, which equals (if we keep 15 digits after the decimal point) to the result of [24], $u=0.12374654323755$. The same confirmation was made with [25]. The axial velocity profiles computed by using the velocity (14) for different values of parameters Ha, α, b and k and have been shown through Figs. 4–15.

Fig. 4–Fig. 8, for unsteady MHD pulsatile flow through porous medium in the presence of magnetic field. Fig. 4 show the relation between axial velocity with time. Fig.5 show that as the Hartmann number increases the axial velocity decreases. Fig. 6 the axial velocity of the blood increases with increasing the permeability parameter of porous medium. Fig. 7and Fig. 8 shows the effect of the Womersley parameter on the axial velocity distribution, we note that the axial velocity increase with increasing Womersley parameter.

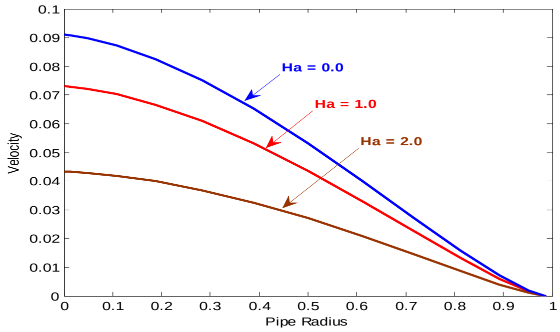

Fig. 10. The effect of Hartman number on velocity distribution [$\alpha=1$, $A_O=1$, $A_1=1$, $a_O=1$, $b=2$, $1/k=2$, $t=1$].

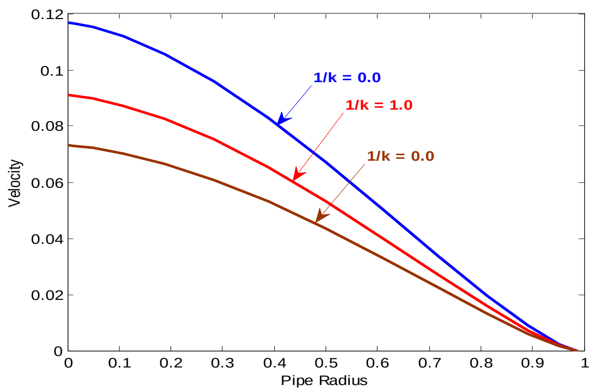

Fig. 11. The effect of porosity number on velocity distribution [$\alpha=1$, $A_O=1$, $A_1=1$, $a_O=1$, $b=2$, $Ha=1$, $t=1$].

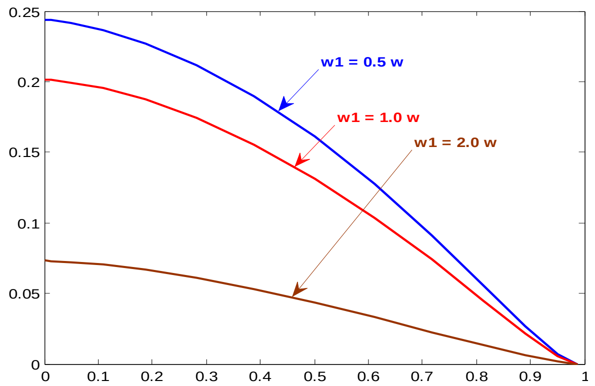

Fig. 12. The effect of frequency of body acceleration on velocity distribution with the pipe radius [$\alpha=1$, $A_O=1$, $A_1=1$, $a_O=1$, $Ha=1$, $1/k=2$ $t=1$].

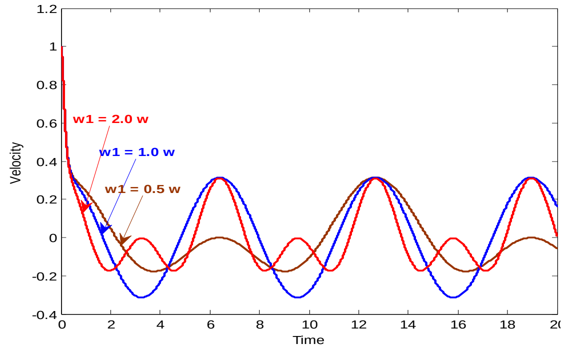

Fig. 13. The effect of frequency of body acceleration on velocity distribution with time [α=1, A_O=1, A_1=1, a_O=1, Ha=1, 1/k=2 t=1].

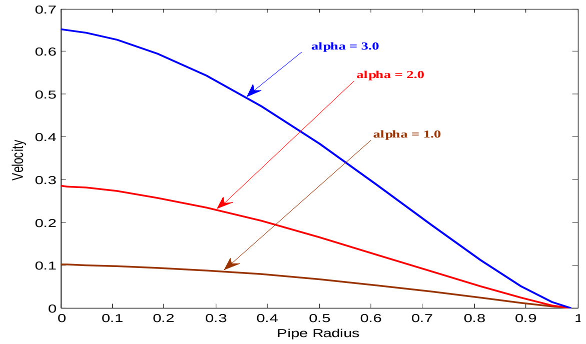

Fig. 14. The effect of Womersley parameter on velocity distribution [A_O=1, A_1=1, a_O=1, b=2, Ha=2, 1/k=1, t=1].

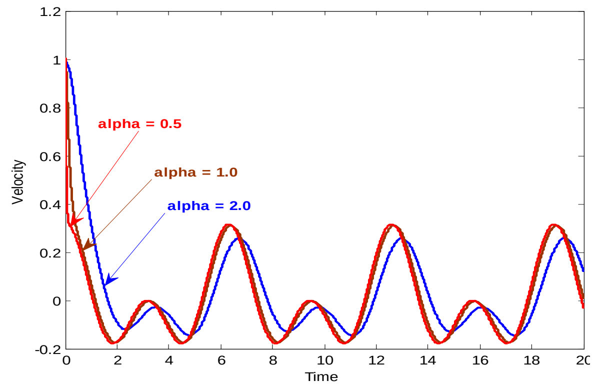

Fig. 15. The effect of Womersley parameter velocity distribution with time [A_O=1, A_1=1, a_O=1, b=2, Ha=1, 1/k=2 t=1].

Fig. 9–Fig. 14 for unsteady MHD pulsatile flow through porous medium in the presence of magnetic field under the effect of body acceleration, Fig.9 shows the relation between velocity distributions with time, Fig.10 show that as the Hartmann number increases the axial velocity decreases. Fig.11 the axial velocity of the blood increases with increasing the permeability parameter of porous medium. Fig. 12 and Fig. 13 shows the effect of the frequency of the body acceleration on the axial velocity distribution, we note that the axial velocity decreases with increasing the frequency of body acceleration. Fig. 14 and Fig. 15 shows the effect of the Womersley parameter on the axial velocity distribution, we note that the axial velocity increase with increasing Womersley parameter.

VI. Conclusions

In the present mathematical model, the unsteady pulsatile blood flow through porous medium in the presence of magnetic field with periodic body acceleration through a rigid straight circular tube (artery) has been studied. The velocity expression has been obtained in numerically. It is of interest to note that the axial velocity increases with increasing of the permeability parameter of porous medium and Womersley parameter whereas it decreases with increasing the Hartmann number, frequency of body acceleration.

The present model gives a numerical solution of velocity distribution with pipe radius and time. It is of interest to note that the result of the present model includes results of different mathematical models such as: The results of [24], [25], the results of [23] have been recovered by taking the permeability of porous medium $k \to \infty$ without stochastic and no body acceleration, the results of [19] have been recovered by taking Hartmann number $Ha=0.0$ (no magnetic field), the results of [18] have been recovered by taking the permeability of porous medium $k \to \infty$ and Hartmann number $Ha=0.0$ (no magnetic field).

It is possible that a proper understanding of interactions of frequency body acceleration with blood flow may lead to a therapeutic use of controlled body acceleration. It is therefore desirable to analyze the effects of different types of vibrations on different parts of the body. Such a knowledge of body acceleration could be useful in the diagnosis and therapeutic treatment of some health problems (joint pain, vision loss, and vascular disorder), to better design of protective pads and machines.

By using an appropriate magnetic field it is possible to control blood pressure and also it is effective for conditions such as poor circulation, travel sickness, pain, headaches, muscle sprains, strains, and joint pains.

Hoping that this investigation may have for further studies in the field of medical research, the application of magnetic field for the treatment of certain cardiovascular diseases, and also the results of this analysis can be applied to the pathological situations of blood flow in coronary arteries when fatty plaques of cholesterol and artery clogging blood clots are formed in the lumen of the coronary artery. The study is useful for evaluating the role of porosity when the body is subjected to magnetic resonance imaging (MRI).

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