Controlling a VTOL in 2-DOF Subspaces

Aydin Yesildirek and Bara Emran

***Abstract*—A switching controller for vertical take off and landing (VTOL) vehicle is presented. Since the entire system belongs to a class of underactuated systems, it is decomposed in to three fully-actuated subsystems. Then, a set of constraints on the rotor speeds are given to cancel out some nonlinearities in input, hence ensure to obtain an affine in control dynamics that enables us to derive local feedback linearizing controllers and together with some associated Lyapunov functions. Finally, a controller scheduler based on Lyapunov functions is invoked to select the subsystem to be controlled and achieve the desired tracking motion. An illustration is given to show the effectiveness of the controller.**

***Index Terms*—Quadrotor, lyapunov function, underactuated systems ,VTOL.**

I. Introduction

We face underactuated systems in a wide range of industrial applications varying from aerospace to robotics, flexible to mobile platforms. They may be formed due to some number of reasons such as the lack of sufficient actuators that may exist because of some cost constraints, some physical difficulties or limitations, in some cases unexpected actuator failures. However, when a system is intentionally built to operate as an underactuated system, we assume that it is controllable in the region of intended operations. In other words, there exists a feasible operation in the configuration space that is achievable using the selected actuation mechanism. For instance, using four rotors, it is possible to fly a quadrotor for surveillance purposes.

VTOL motion may certainly be obtained by using four-rotor helicopters, quadrotors, which are studied extensively in literature for many applications due to their low-cost, maintainability, and maneuverability features. Quadrotors, having only four rotors, do not have enough control richness in their structure to provide unrestricted flight in full vector space. Hence, the quadrotors are not fully linearizable. In this paper, we will consider tracking problem of a desired linear motion in the Cartesian space together with the heading angle.

There exist several approaches to obtain a suitable controller for this class underactuated systems. First solutions have been shown on linearized dynamical models around some predefined flight conditions. Classical PID algorithms have been demonstrated successfully due to their simplicity and proven reliability in practice. Multi-loop PID architectures derived for a particular flight profile over a trimmed model have been shown to perform well. For instance, a PID controller is derived and compared to LQR and backstepping techniques in [1]. In [2] a gain scheduled PID technique is applied to quadrotor with some fault conditions. There is an example of a sliding mode controller for altitude control is shown [3]. Effectiveness of backstepping under disturbance for attitude control is shown in [4]. An H_∞ control application is shown in [5], dynamic inversion based control scheme is given in [6]. In [7], an adaptive sliding mode and a feedback linearization methods are compared for the quadrotor platform. A nonlinear controller for a simplified model of quadrotor using a control Lyapunov function (clf) [8] has been shown in [9].

Our focus is to develop a novel structure to enable us to control the linear motion and the heading angle of quadrotor.

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II. Background

A. Underactuated Dynamical Systems

Rigid body dynamics may be derived using Euler-Lagrange method will have the following general equations of motion.

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d = \tau \qquad (1)$$

with q configuration space, M , V_m some matrices $G(q)$ and $F(\dot{q})$ some vectors. Bounded unknown disturbances and the control input torque are denoted by τ_d and τ respectively. Underactuated systems are defined when the system DOF is larger than the number of actuators. For such systems, one can partition the configuration space $x = [x_a \ x_u] \in \mathrm{R}^n$ and rewrite the (1) as

$$\frac{d}{dt}\begin{bmatrix} x_a \\ x_u \end{bmatrix} = \begin{bmatrix} f_a \\ f_u \end{bmatrix} + \begin{bmatrix} g_a \\ g_u \end{bmatrix} u \qquad (2)$$

with $x_a \in \Omega_a$ actuated and $x_u \in \Omega_u$ unactuated vector spaces, $u \in \mathrm{R}^{n_a}$, f_i, g_i for $i = a,u$ smooth vector fields in appropriate dimensions.

B. Partial Feedback Linearization

Reference [10] has shown the existence of a collocated controller

$$u = (g_a^T g_a)^{-1}\left[-f_a(x) + \nu\right] \qquad (3)$$

with ν auxiliary control input that partially feedback linearizes underactuated system in (2) as

$$\dot{x}_a = \nu \qquad (4)$$

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$$\dot{x}_u = \alpha_u(x) + \beta_u(x)v \tag{5}$$

with $\alpha_u, \beta_u \in \mathrm{R}^{n_u}$ uncontrolled internal dynamics nonlinearities. When a mechanism is intentionally built as an underactuated system, it is understood that there exists a sufficient controller within the region of operation.

To extend the similar discussion in to the uncollocated space let us repartition x_u into n_a-DOF subsystems. For an n-DOF system, let $N = \left\lceil \frac{n}{n_a} \right\rceil$. Then, (5) may be generalized as

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix} u \tag{6}$$

If all $g_i g_i^T(x)$ are full rank matrices almost everywhere for $i = 1,2,\cdots N$ then there exists N controllers

$$u_i = g_i^T (g_i g_i^T)^{-1}(x)[-f_i(x) + v_i] \tag{7}$$

that partially feedback linearizes (6) in to N subsystems as

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{x}_j &= \alpha_j(x) + \beta_j(x)v_i \quad \forall j \neq i \end{aligned} \tag{8}$$

with $x_i \in \Omega_i \subset \mathrm{R}^{n_a}$ sub-vectors. For such systems, the v_i can only be designed to control the actuated i^{th} subsystem with n_a-DOF. Without losing generality we can assume n is a multiple of n_a.

Assumption 1: Any one of the N subsystem is controllable when the entire system is stable within the region of operation. Although it may seem to be strong assumption, it is expected for the systems where the actuation mechanism is designed to be less than full DOF. This is also applicable for VTOLs.

C. *Switching Lyapunov Functions*

Let us try to formulate a dynamic representation of class of nonlinear systems under N-switching inputs as follows

$$\dot{x}(t) = f(x, u_i(x); t) = \zeta_i(x; t) \tag{9}$$

with $x \in \Omega \subset \mathrm{R}^n$, $f(x, u_i; t) = \zeta_i$ Lipschitz continuous vector fields, $i \in \ell; \{1, \cdots, N\}$. Without loosing generality $\zeta_i(0) = 0$ for all $i \in \ell$.

Let us, now, define a piecewise constant switching signal $\sigma(t): [0, \infty) \to \ell$. Assume that there is a finite number of switching instances in any finite interval defined by a sequence $T_S = \{t_0, t_1, \cdots\}$.

Let us define Lyapunov-like function first.

Definition 1: A system has Lyapunov-like function V in Ω_x if

1) There exists $\alpha_2 > \alpha_1 > 0$ such that

$$\alpha_1 \|x\|^2 < V < \alpha_2 \|x\|^2 \quad \forall x \in \Omega_x$$

2) Its derivative is negative definite within the piece-wise continues time intervals $[t_k, t_{k+1})$

$$\dot{V}(x) < 0 \quad \forall x \in \Omega_x \setminus \{x = 0\} \quad and \quad \dot{V}(0) = 0$$

3) It forms a decreasing sequence at the switching instances of T_S

$$V(x(t_{k+1}); t_{k+1}) < V(x(t_k); t_k) \quad for\ t_{k+1} > t_k$$

Theorem 1 ([11]): Given the N-switched system (9), suppose each vector field ζ_i has an associated Lyapunov-like function V_i in the region Ω_i with equilibrium point $\bar{x} = 0$, let $\sigma(t)$ be a switching sequence such that $\sigma(t)$ can take on the value i only if $x(t) \in \Omega_i$, and in addition

$$V_i(x(t_{i,k})) \leq V_i(x(t_{i,k-1})) \tag{10}$$

where $t_{i,k}$ denotes the k th time that ζ_i is switched in. Then (9) is Lyapunov stable.

III. Problem Definition

A quadrotor as a VTOL device having with four rotors is shown in Fig. 1. Controlling the entire 6-DOF in VTOL with only four actuators is not possible.

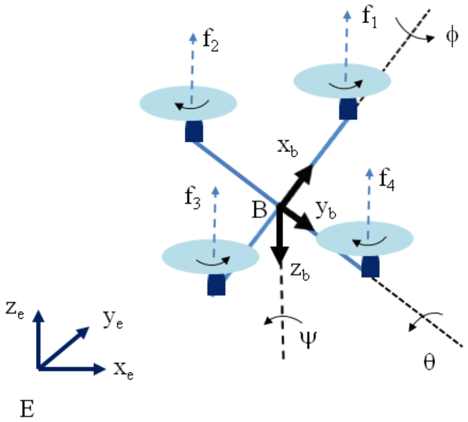

Fig. 1. Quadrotor frame.

Controlling the entire 6-DOF in VTOL with only four actuators is not possible. This structure is built in merely for the purpose of navigation in 3D Cartesian space with vertical takeoff and landing characteristics. Therefore, we consider that it is a functional underactuated system within the scope of this intended region of operation.

A. *VTOL Dynamics*

Relevant coordinate frames are shown in Fig. 1 as the earth-frame, E, is a fixed reference frame and the body frame, B, attached to the vehicle. An Euler-Lagrange based model is given in [6] as

$$\begin{aligned}\ddot{x} &= \frac{-\sin(\theta)}{m}u \\ \ddot{\theta} &= \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_p}{I_y}\dot{\phi}\Omega + \frac{l}{I_y}\tau_\theta\end{aligned} \tag{11}$$

$$\begin{aligned}\ddot{y} &= \frac{\cos(\theta)\sin(\phi)}{m}u \\ \ddot{\phi} &= \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_p}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}\tau_\phi\end{aligned} \tag{12}$$

$$\begin{aligned}\ddot{z} &= -g + \frac{\cos(\theta)\cos(\phi)}{m}u \\ \ddot{\psi} &= \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{l}{I_z}\tau_\psi\end{aligned} \tag{13}$$

with inputs, lift force

$$u = f_1 + f_2 + f_3 + f_4,$$

and generalized torques

$$\tau_a = \begin{bmatrix}\tau_\phi \\ \tau_\theta \\ \tau_\psi\end{bmatrix} = \begin{bmatrix} l(f_3 - f_1) \\ l(f_2 - f_4) \\ c(f_1 - f_2 + f_3 - f_4)\end{bmatrix},$$

The thrusts generated by rotor driven propellers may be shown as

$$f_i = k\omega_i^2$$

where k is the lift constant, ω_i angular speed and the mismatch speed term

$$\Omega = \omega_2 + \omega_4 - \omega_1 - \omega_3.$$

B. Input Constraints

The system dynamics (11)-(13) are not in the affine form. To transform the nonlinear control input terms, we constrain the angular velocities that generate lift force and generalized torques according to (14).

	w_1	w_2	w_3	w_4
pitch	ω_0	$\omega_0 + \Delta$	ω_0	$\omega_0 - \Delta$
roll	$\omega_0 - \Delta$	ω_0	$\omega_0 + \Delta$	ω_0
yaw	$\omega_0 + \Delta$	$\omega_0 - \Delta$	$\omega_0 + \Delta$	$\omega_0 + \Delta$

(14)

Note that at the expense of reducing the effective control inputs to the already underactuated system we gain a set of equations of motion in the affine form. Let us define three subsystems to control 2-DOF motion using the nominal speed ω and the speed variation Δ needed to achieve the desired motion. Hence, the system model can be written in the form of (6) where

$$\begin{aligned} x_1 &= \left[x\ \theta\ \dot{x}\ \dot{\theta}\right]^T \in \Omega_p \\ x_2 &= \left[y\ \phi\ \dot{y}\ \dot{\phi}\right]^T \in \Omega_r \\ x_3 &= \left[z\ \psi\ \dot{z}\ \dot{\psi}\right]^T \in \Omega_y \end{aligned} \tag{15}$$

$\Omega_i \subset \mathrm{R}^4$ for $i = p, r, y$ defining pitch, roll and yaw subsystems respectively.

Required forces needed for the lateral motion can be obtained by small rotations around θ and ϕ angles. Therefore, to operate the VTOL it is sufficient to keep the angles bounded by

$$-\frac{\pi}{2} < \phi < \frac{\pi}{2}, \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

C. Error System Dynamics in Fully-Actuated Subspaces

Let there be a piecewise continuous desired motion defined by the vector $q_d = [x_d\ y_d\ z_d\ \psi_d]^T$. Due to the underactuation constraints we relax the roll and pitch motion for the tracking problem. Let us study the tracking error in each subsystem.

1) Pitch subsystem

Let the position tracking error in pitch subspace be defined as

$$e_p = \begin{bmatrix} e_x \\ e_\theta \end{bmatrix} = \begin{bmatrix} x_d - x \\ \theta_d - \theta \end{bmatrix} \tag{16}$$

Note that θ_d is a virtual desired signal utilized to provide the motion in x-dimension. Now, define a filtered tracking error as

$$s_p = \dot{e}_p + \Lambda_p e_p \tag{17}$$

with $\Lambda_p = diag\{\lambda_x, \lambda_\theta\} > 0$. Its dynamics can be expressed by taking derivative and using (11) and (14) as

$$\dot{s}_p = \begin{bmatrix} \ddot{x}_d + \lambda_x \dot{e}_x \\ \ddot{\theta}_d + \lambda_\theta \dot{e}_\theta - \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right)\end{bmatrix} + \begin{bmatrix} \frac{\sin(\theta)}{m} & 0 \\ 0 & -\frac{l}{I_y}\end{bmatrix}\tau_p \tag{18}$$

with $\tau_p = [u\ \tau_\theta]^T$.

2) Roll subsystem

The position tracking error in roll subspace may be defined as

$$e_r = \begin{bmatrix} e_y \\ e_\phi \end{bmatrix} = \begin{bmatrix} y_d - y \\ \phi_d - \phi \end{bmatrix} \tag{19}$$

Now, define filtered tracking errors

$$s_r = \dot{e}_r + \Lambda_r e_r \tag{20}$$

with $\Lambda_r = diag\{\lambda_y, \lambda_\phi\} > 0$. The filtered error dynamics can be expressed by taking derivative of (20) using (12) and (14)

$$\dot{s}_r = \begin{bmatrix} \ddot{y}_d + \lambda_y \dot{e}_y \\ \ddot{\phi}_d + \lambda_\phi \dot{e}_\phi - \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) \end{bmatrix} - \begin{bmatrix} \frac{\cos(\theta)\sin(\phi)}{m} & 0 \\ 0 & \frac{l}{I_x} \end{bmatrix} \tau_r \tag{21}$$

with $\tau_r = [u\ \tau_\phi]^T$.

3) Yaw subspace

Yaw subsystem is where the altitude and vertical takeoff and landing takes place. Therefore the desired position error vector is

$$e_{yaw} = \begin{bmatrix} e_z \\ e_\psi \end{bmatrix} = \begin{bmatrix} z_d - z \\ \psi_d - \psi \end{bmatrix} \tag{22}$$

Now, define the filtered tracking error vector

$$s_y = \dot{e}_{yaw} + \Lambda_y e_{yaw} \tag{23}$$

with $\Lambda_y = diag\{\lambda_z, \lambda_\psi\} > 0$. The filtered error dynamics can be expressed by taking derivative of (22), using (13) and the constraints of inputs as in (14)

$$\dot{s}_y = \begin{bmatrix} \ddot{z}_d + \lambda_z \dot{e}_z - g \\ \ddot{\psi}_d + \lambda_\psi \dot{e}_\psi - \dot{\theta}\dot{\phi}\left(\frac{I_x - I_y}{I_z}\right) \end{bmatrix} + \begin{bmatrix} \frac{\cos(\theta)\cos(\phi)}{m} & 0 \\ 0 & -\frac{l}{I_z} \end{bmatrix} \tau_y \tag{24}$$

with $\tau_y = [u\ \tau_\psi]^T$.

IV. Controller Development

We will give subsystem controllers by the following Lemmas in the flight regimen. Then, define a control scheduler to select the desired fully-actuated subspace using Lyapunov functions.

A. Fully-Actuated Controllers

Desired flight regimen is achieved in one of the three subsystem operations, pitch, roll or yaw. For each subsystem, let's define three controllers in the structure of (7) by the following lemmas.

Theorem 2 (Pitch): Let controller for the pitch domain actuation be defined by the speeds

$$\begin{aligned} \omega_o &= \frac{1}{4\sqrt{k}}(\sqrt{A_p} + \sqrt{B_p}) \\ \Delta &= \frac{1}{2\sqrt{2k}}(\sqrt{A_p} - \sqrt{B_p}) \end{aligned} \tag{25}$$

with

$$\begin{aligned} A_p &= u_p + \frac{\sqrt{2}}{l}\tau_\theta \\ B_p &= u_p - \frac{\sqrt{2}}{l}\tau_\theta \end{aligned} \tag{26}$$

and

$$\begin{aligned} u_p &= \frac{m}{\sin(\theta)}\left[\ddot{x}_d + \lambda_x \dot{e}_x + k_{v_x} s_x\right], \qquad \theta \neq 0 \\ \tau_\theta &= \frac{I_y}{l}\left[\ddot{\theta}_d + \lambda_\theta \dot{e}_\theta - \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) + k_{v_\theta} s_\theta\right] \end{aligned} \tag{27}$$

with $k_{v_y} > 0$ and with $k_{v_\phi} > 0$. Then, the tracking error e_p goes to zero exponentially in $\Omega_p \setminus \{\theta = 0\}$.

Proof: Take a Lyapunov function

$$V_p = \frac{1}{2} s_p^T P_p s_p > 0 \tag{28}$$

with $s_p = \begin{bmatrix} s_x & s_\theta \end{bmatrix}^T$ and $P_p = P_p^T > 0$. The error system dynamic given in (18) with the control (27) yields

$$\dot{s}_p = -K_{v_p} s_p \tag{29}$$

with

$$K_{v_p} = \begin{bmatrix} k_{v_x} & 0 \\ 0 & k_{v_\theta} \end{bmatrix} \tag{30}$$

Thus, the time derivative of (28)

$$\dot{V}_p = -s_p P_p K_{v_p} s_p = -\alpha_p \|s_p\|^2 < 0 \tag{31}$$

Therefore s_p goes to zero exponentially. Using the standard Lyapunov theorem one can conclude that e goes to zero and using the Barbalat's extension so does $\dot{e}$. Hence the tracking is achieved in $\Omega_p \setminus \{\theta = 0\}$.

Theorem 3 (Roll): Let the roll subspace dynamics be controlled by the speeds of

$$\begin{aligned} \omega_o &= \frac{1}{4\sqrt{k}}(\sqrt{A_r} + \sqrt{B_r}) \\ \Delta &= \frac{1}{2\sqrt{2k}}(\sqrt{A_r} - \sqrt{B_r}) \end{aligned} \tag{32}$$

with

$$\begin{aligned} A_r &= u_r + \frac{\sqrt{2}}{l}\tau_\phi \\ B_r &= u_r - \frac{\sqrt{2}}{l}\tau_\phi \end{aligned} \tag{33}$$

with

$$\begin{aligned} u_r &= \frac{m}{\cos(\theta)\sin(\phi)}\left[\ddot{y}_d + \lambda_y \dot{e}_y + k_{v_y} s_y\right], \qquad \phi \neq 0 \\ \tau_\phi &= \frac{I_x}{l}\left[\ddot{\phi}_d + \lambda_\phi \dot{e}_\phi - \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) + k_{v_\phi} s_\phi\right] \end{aligned} \tag{34}$$

with $k_{v_y}, k_{v_\phi} > 0$. Then, e_2 goes to zero exponentially in $\Omega_r \setminus \{\phi = 0\}$.

Proof: Take a Lyapunov function

$$V_r = \frac{1}{2} s_r^T P_r s_r \tag{35}$$

with $s_r = [s_y\ \ s_\phi]^T$ and $P_r = P_r^T > 0$. Substituting the proposed controller (34) in to (21) results in

$$\dot{s}_r = -K_{v_r} s_r \tag{36}$$

with K_{v_r} positive diagonal matrix as in (30). Thus, the time derivative of (35)

$$\dot{V}_r = -s_r^T P_r K_{v_r} s_r = -\alpha_r \|s_r\|^2 < 0 \tag{37}$$

Similar to pitch subspace, the exponentially convergence to zero tracking error is achieved in $\Omega_r \setminus \{\phi = 0\}$.

Theorem 4 (Yaw): Finally, controller for the yaw subspace is constructed by

$$\begin{aligned} \omega_o &= \frac{1}{4\sqrt{k}}\left(\sqrt{A_y} + \sqrt{B_y}\right) \\ \Delta &= \frac{1}{4\sqrt{k}}\left(\sqrt{A_y} - \sqrt{B_y}\right) \end{aligned} \tag{38}$$

with

$$\begin{aligned} A_y &= u_y + \frac{1}{c}\tau_\psi \\ B_y &= u_y - \frac{1}{c}\tau_\psi \end{aligned} \tag{39}$$

with

$$\begin{aligned} u_y &= \frac{m}{\cos(\theta)\cos(\phi)}\left[\ddot{z}_d + \lambda_z \dot{e}_z - g + k_{v_z} s_z\right] \\ \tau_\psi &= \frac{I_z}{l}\left[\ddot{\psi}_d + \lambda_\psi \dot{e}_\psi - \dot{\phi}\dot{\theta}\left(\frac{I_x - I_x}{I_z}\right) + k_{v_\psi} s_\psi\right] \end{aligned} \tag{40}$$

with $k_{v_z}, k_{v_\psi} > 0$. Then, $[z\, \psi\, \dot{z}\, \dot{\psi}]^T$ goes to $[z_d\, \psi_d\, \dot{z}_d\, \dot{\psi}_d]^T$ exponentially in Ω_y.

Proof: Take a Lyapunov function

$$V_y = \frac{1}{2} s_y^T P_y s_y > 0 \tag{41}$$

with

$$s_y = [s_z\ \gamma s_\psi]^T \tag{42}$$

Substituting the proposed controller (40) in (24) and having the time derivative of (42) result in

$$\dot{s}_y = -K_{v_y} s_y \tag{43}$$

Thus, the time derivative of (41)

$$\dot{V}_y = -s_y P_y K_{v_y} s_y = -\alpha_y P s_y P^2 < 0 \tag{44}$$

Once again, having s_y go to zero exponentially that implies that the tracking objective is achieved everywhere in Ω_y.

B. Main Result

So far, we have derived three fully-actuated controllers for 2-DOF subsystems in three flight conditions. We, now, show a scheduler to select the proper controller for the subsystem needing to be controlled.

Theorem 5: Let the pair (τ_i, V_i) with $i = p, r, y$ define the fully-actuated subsystem controllers and the corresponding clf. Then, the control scheduler using

$$u = \{u_i : V_i = \max\{V_p, V_r, V_y\}\} \tag{45}$$

guarantees the tracking of the desired quadrotor motion in the sense of Lyapunov.

Let us consider

$$V_\sigma = \max\{V_p, V_r, V_y\} \tag{46}$$

This is a bounded function, since

$$a_l^i \|x\| < V_i \le a_u^i \|x\|$$

with $i = p, r, y$, then

$$\underline{a}\|x\| \le V_\sigma \le \bar{a}\|x\|$$

with

$$\underline{a} = \max\{a_l^i\}, \quad \bar{a} = \min\{a_l^i\} \quad \forall i.$$

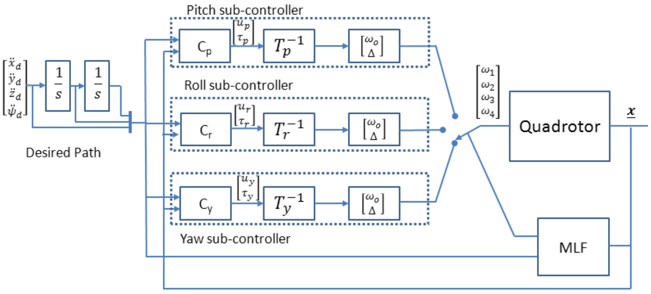

Fig. 2. Proposed control framework.

It is a piecewise decreasing function in the intervals defined by $\sigma(t)$. We have from Lemma 1-3 that at each interval, the selected control u_i ensures the exponential decay of V_i.

Due to the continuity of V_i for $i = p, r, y$, the function satisfies $V_{\sigma(t_{i+1})} < V_{\sigma(t_i)}$ since V_i's cannot jump at the time of switching instants.

Therefore, V_σ is a Lyapunov-like function. Invoking the Theorem 1 we conclude that the systemm is Lyapunov stable. This concludes the proof.

V. Illustrative Example

To demonstrate the proposed controller we pick a small quadrotor with mass of $m = 1\ kg$, arm length $l = 0.24m$, moments of inertia about body frame axis $I_{xx} = I_{yy} = 8.1 \cdot 10^{-3}\ kgm^2$, $I_{zz} = 14.2 \cdot 10^{-3}\ kgm^2$, motor inertia $J = 104 \cdot 10^{-8}\ kgm^2$, thrust factor $k = 54.2 \cdot 10^{-6}\ Ns^2$, drag factor $d = 1.1 \cdot 10^{-6}\ Nms^2$. The limits on the motor actuators are set to be $500\ rad/s$.

TABLE I: THE DESIRED TRAJECTORY

Time	Desired path
$[0, T_1)$	$(x_o, y_o, \frac{z_1}{T_1} t, 0)$
$[T_1, T_2)$	$(x_o, y_o, z_1, \frac{\psi_1}{T_2 - T_1}(t - T_1)$
$[T_2, T_3)$	$(x_o + R\sin(w(t - T_2)), y_o - R + R\cos(w(t - T_2)), z_1, \psi_1)$
$[T_3, T_4)$	$(x_o + \frac{x_1 - x_o}{T_2}(t - 3T_2), y_o + \frac{y_1 - y_o}{T_2}(t - 3T_2), z_1, \psi_1)$

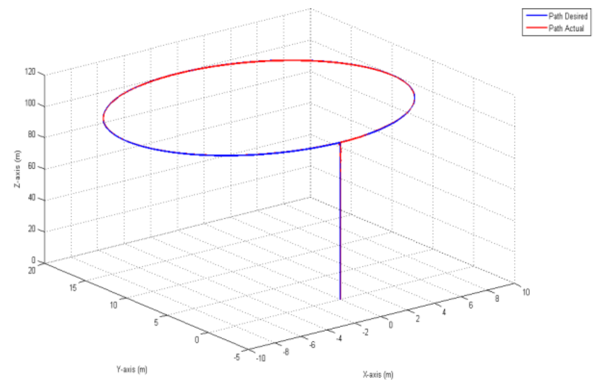

Fig. 3. Desired and actual paths.

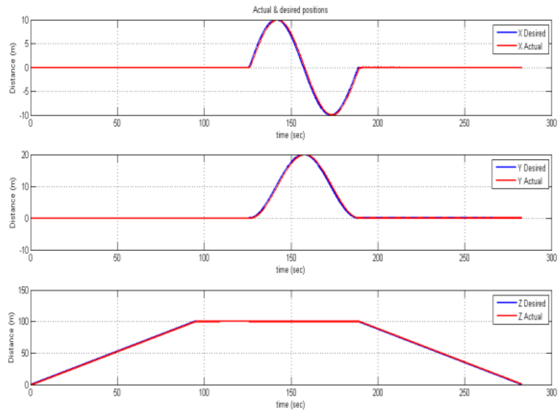

Fig. 4. Desired and actual positions.

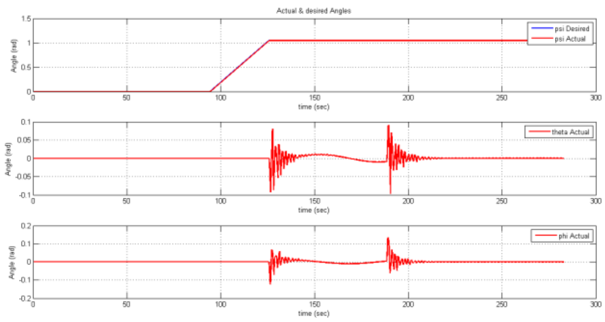

Fig. 5. Actual and desired angles.

The desired trajectory is defined by vertical takeoff, hover, circle and then vertical land by the trajectory as Control parameters and constructed by the sliding surface constants $\lambda_i = 30$, control gain matrices $K_{v_i} = 30I$, Lyapunov function matrices $P_i = 30I$. Simulation results show the actual and desired 3D paths in Fig. 3. More detailed navigational errors are shown in Fig. 4 and Fig. 5.

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