The Study of the Natural Convection in an Infinite Horizontal Channel Partially Heated by Lattice Boltzmann Method

S. Houat and N. Saidi

***Abstract*—In this paper, a lattice BGK model for thermal flows is used to simulate laminar natural convection in an infinite horizontal channel, partially heated by Lattice Boltzmann method. We applied the model based on the double-distribution function approach in two dimensional. Velocity and temperature distributions as well as Nusselt number were obtained and analyzed for the Rayleigh number ranging from 2.10^3 to 5.10^4 with the Prandtl number equal 0.71.**

***Index Terms*—Double distribution function LBGK, horizontal channel, natural convection, thermal lattice Boltzmann method.**

I. Introduction

Natural convection represents an extremely interesting subject due to the coupling between fluid flow and energy transport. This area of research is attractive because of its potential in practical engineering applications such as thermal design of buildings, furnace design, electronic equipment and others. Previously, many investigators have studied convection in various geometries. The development of theoretical models, numerical algorithms and experimental approaches constitutes a solid base for advancement of knowledge in this field.

The lattice Bhatnagar–Gross–Krook (LBGK) method, a novel kinetic-based numerical approach for simulating fluid flows and associated transport phenomena, has developed rapidly since its emergence. Many studies have made great strides in constructing its theoretical foundation [1]-[4] and improving its numerical performance [5]–[9] over the last decade. Unlike conventional numerical schemes, which discretize the macroscopic governing equations directly, the LBGK method solves the kinetic equation at the mesoscopic scale, i.e. the Boltzmann equation with the BGK assumption [1], [2]. Historically, the LBGK method originated from the lattice-gas cellular automata method (LGCA) [8]-[10], a microscopic model for fluid systems where the imaged fluid particles collide and move on a regular lattice, and indeed it is very similar to the LGA method, except that particles residing on the lattice are replaced by the corresponding distribution functions and the collision operator is approximated by the BGK assumption. But later it was realized that LBGK could also be viewed as a special finite difference scheme of the continuous Boltzmann equation on a regular lattice [1], [2], which also defines the associated discrete particle velocities. From this viewpoint, discretization for the particle velocity can be decoupled from the spatial discretization, since the particle velocity in the Boltzmann equation is independent of the particle position [5]. This implies that we can discretize the continuous velocity space into a set of discrete velocities with sufficient symmetry (physical symmetry).

Currently, a few thermal lattice Boltzmann models have been proposed. The earliest model which is known as multi speed model [11], uses the same distribution function in defining the macroscopic temperature. However, this model is reported to suffer numerical instability [12] and has a demerit that it can simulate thermal fluid flows only at fixed Prandtl number [13]. As an alternative approach, Shan proposed the so-called passive-scalar model [14]. This model suggests that the flow fields (velocity and density) and the temperature are represented by two different distribution functions. The macroscopic temperature is assumed to satisfy the same evolution equation as a passive scale, which is advected by the flow velocity but does not affect the flow field.

The work of Luo and He [15] demonstrated that the isothermal lattice Boltzmann equation can be directly obtained by properly discretizing the continuous Boltzmann equation in both time and space phases. Following the same procedure, He [16] proposed the double-distribution function model, where the thermal lattice Boltzmann evolution equation can be derived by discretizing the continuous Boltzmann equation for the internal energy distribution. It has been shown that this model is simple and applicable to problems with different Prandtl numbers [17]. More importantly, this model requires low order moment and thus provides higher numerical stability than the passive-scalar model.

In this paper, we use two dimensional simulation for natural convection heat transfer in an infinite horizontal channel partially heated developed new five velocity lattice models of the internal energy density distribution function for incompressible flow.

II. Description of Physical Problem

The studied configuration, sketched in Fig. 1, is an infinite channel discretely heated from below where adiabatic partitions are regularly placed at the center of the adiabatic

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surfaces. The lower wall is partially heated and maintained at constant temperature at $T2=1$. The remaining portions of the lower boundary are adiabatic. The upper wall of the channel, placed at a height H from the lower one, is also maintained at constant but cold temperature at $T1=0$.

The periodic nature of the system and the associated boundary conditions permits the subdivision of the channel into finite simple domains (SD) of length L. The study can be conducted in a SD, limited by the fictive boundaries P1 and P2 (Fig. 1). Such technique of subdivision was used in the past by [18] and [19] to study the natural convection in a channel provided, respectively, with adiabatic and heated portions on its lower wall. In references [19] and [20] have shown that although the SD is a representative entity of the studied configuration, its use allows obtaining only solutions verifying the periodic conditions imposed by P1 and P2. They have, thus, established the limitations of the SD by considering a calculation domain twice as long and called a double domain (DD) of length 2L. Consequently, in the present study, the simulations were performed in a DD, limited by the space between the fictive boundaries P1 and P3.

III. Numerical Method

A. Double Population Thermal Lattice Boltzmann Method

The physical space is divided into a regular lattice and the velocity space is discretized into a finite set of velocities $\{c_\alpha\}$, the Boltzmann equation with Bhatnagar-Gross-Krook (BGK) approximation [21] can be discretized as [17], [22]:

$$f_\alpha(x_i + c_\alpha \Delta t, t+\Delta t) - f_\alpha(x_i,t) = -\frac{f_\alpha(x_i,t) - f_\alpha^{eq}(x_i,t)}{\tau_v} + \Delta t.F_\alpha \quad (1)$$

where, Δt and $\boldsymbol{c}_\alpha \Delta t$ are time and space increments, respectively. f_α is the single-particle velocity distribution function along the α^{th} direction. f_α^{eq} is the equilibrium distribution function, τ_v is the single relaxation time and F_α is the external force.

There are different types of lattice for LBM. For simplicity and without loss of generality, we consider the two-dimensional square lattice with nine velocities, the D2Q9 model:

$$c_\alpha = ce_\alpha = \begin{cases} (0,0) & \alpha = 0, \\ c[\cos(\frac{\alpha-1}{2}\pi), \sin(\frac{\alpha-1}{2}\pi)] & \alpha = 1,2,3,4, \\ \sqrt{2}c[\cos(\frac{\alpha-5}{2}\pi+\frac{\pi}{4}), \sin(\frac{\alpha-5}{2}\pi+\frac{\pi}{4})] & \alpha = 5,6,7,8, \end{cases} \quad (2)$$

The equilibrium distribution function for D2Q9 model is given by:

$$f_\alpha^{eq} = \rho w_\alpha \left[1 + \frac{3}{c^2} c_\alpha \cdot u + \frac{9}{2c^4}(c_\alpha \cdot u)^2 - \frac{3}{2c^2} u \cdot u\right] \quad (3)$$

where $w_0 = 4/9$, $w_1 = w_2 = w_3 = w_4 = 1/9$, and $w_5 = w_6 = w_7 = w_8 = 1/36$. The macroscopic density ρ and velocity $\boldsymbol{u}$ are related to the distribution function by:

$$\rho = \sum_{\alpha=1}^{9} f_\alpha \quad (4)$$

$$\rho u = \sum_{\alpha=1}^{9} f_\alpha c_\alpha \quad (5)$$

Using the Chapman-Enskog expansion, the equation (1) can recover the Navier-Stokes equation to the second order of accuracy, with the kinematic viscosity given by:

$$v = \frac{(\tau_v - 0.5)c^2 \Delta t}{3} \quad (6)$$

The LBGK evolution equation for the temperature is [17]:

$$g_\alpha(x_i + c_\alpha \Delta t, t+\Delta t) - g_\alpha(x_i,t) = -\frac{g_\alpha(x_i,t) - g_\alpha^{eq}(x_i,t)}{\tau_T} \quad (7)$$

For the evolution of g_α, given its simplified equilibrium distribution function, a D2Q5 lattice is preferred [21]. In the D2Q5 topology, the velocities v_α are:

$$v_\alpha = \begin{cases} (0,0) & \alpha = 0, \\ c[\cos(\frac{\alpha-1}{2}\pi), \sin(\frac{\alpha-1}{2}\pi)] & \alpha = 1,2,3,4, \end{cases} \quad (8)$$

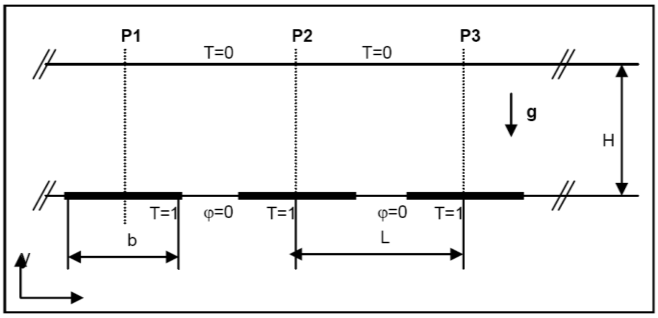

Fig. 1. Geometry of physical problem.

The associated weights w^T_α are $w^T_0 = 1/3$, $w^T_1 = w^T_2 = w^T_3 = w^T_4 = 1/6$. The equilibrium distribution function for D2Q5 model is given by:

$$g_\alpha^{eq} = T\, w_\alpha^T \left[1 + \frac{3}{c^2} v_\alpha \cdot u\right] \quad (9)$$

At each lattice node, the macroscopic temperature is defined as:

$$T = \sum_{\alpha=1}^{5} g_\alpha \quad (10)$$

And the thermal diffusivity (in lattice units) is related to the relaxation time is :

$$\kappa = \frac{(\tau_T - 0.5)c^2 \Delta t}{3} \quad (11)$$

B. Heat Transfer

The thermal conditions applied on the two parallel stationary walls, the hot (bottom) and the cold (top) introduces a temperature gradient in a fluid, and the consequent density difference what induces a fluid motion that is, convection. In the simulation, the Boussinesq approximation is applied to the buoyancy force term [14], [16], [17], [22]:

$$\rho G = \rho\beta g_0 \left(T - T_0\right) j \tag{12}$$

where β is the thermal expansion coefficient, g_0 is the acceleration due to gravity, T_0 is the average temperature and j is the vertical direction opposite to that of gravity. So the external force in Eq. (1) is

$$F_f = 3G(c-u)f^{eq} \tag{13}$$

The dynamical similarity depends on two dimensionless parameters: the Prandtl number, *Pr* and the Rayleigh number, *Ra* defined as,

$$Pr = \frac{\nu}{\kappa} \quad \text{and} \quad Ra = \frac{g_0 \beta \Delta T H^3}{\nu\kappa} \tag{14}$$

where ΔT is the wall temperature difference, *H* is the distance between the walls.

The Nusselt number, *Nu* is one of the most important dimensionless numbers in describing the convective transport. The Nusselt number for the hot wall is defined as the ratio between the heat transports by convection to the heat transmission due to conduction:

$$Nu = 1 + \frac{\langle u_y T \rangle H}{\kappa \Delta T} \tag{15}$$

Here $\langle u_y T \rangle$ denotes the average over the convection layer.

IV. Results and Discussions

This kind of the study is a classical benchmark on the thermal models defined by The Rayleigh-Benard convection flow. The fluid is enclosed between two parallel stationary walls, the hot (bottom) and the cold (top), and experiences the gravity force. Density variations caused by the temperature variations drive the flow, while the viscosity will counteract to equilibrate it.

In all our simulations, the study is stationary, the Prandtl number *Pr*=0,71 and Rayleigh number is varied between 2.10^3 and 5.10^4. For the boundary conditions at the top and the bottom walls, we applied the bounce back condition is applied for the fluid distribution. Different numbers of nodes were tested and the results were similar. The results presented here are $Nx \times Ny$ = 121×61 for the periodic side boundary conditions.

A. Validation Model

The configuration used for validation of the model is a horizontal chanel totally heated in bottom wall. The geometry configuration is similar in Fig. 1 except that b/L ratio equal 1 (i.e. 100% heated). After simulation for different Rayleigh numbers, Extrapolating the obtained values of Nusselt number, an estimate of the final converged solution can be done (*Nu*). In Fig. 2, the isotherms of $Ra = 5.10^3$ and $Ra = 10^4$ are plotted for the case of 121 grid nodes in the y-direction. In Fig. 3, the contours of the stream function of the incompressible flow field for $Ra = 5.10^3$ and $Ra = 10^4$ are plotted.

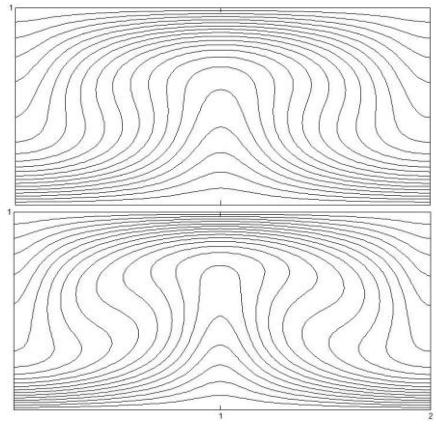

Fig. 2. Contour plot with iso-temperature lines for $Ra = 5.10^3$ (top) and $Ra = 10^4$ (bottom).

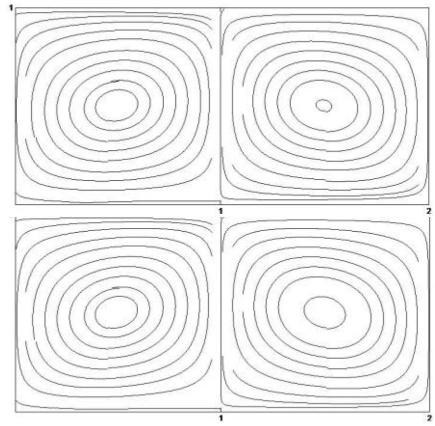

Fig. 3. Contour plot with the stream function for $Ra = 5.10^3$ (top) and $Ra = 10^4$ (bottom).

What showed the appearance of two Benard rollers against rotative with height and width equal *H* (height of the channel). The extrapolated converged values of Nusselt number at various Rayleigh numbers are plotted in Fig. 4, and compared with a empirical power law [23] and the standard reference data [24]. The critical Rayleigh number is defined by Ra_c=1707.8. The present results obtained by TLBM model is found to be in good agreement with [23] and [24].

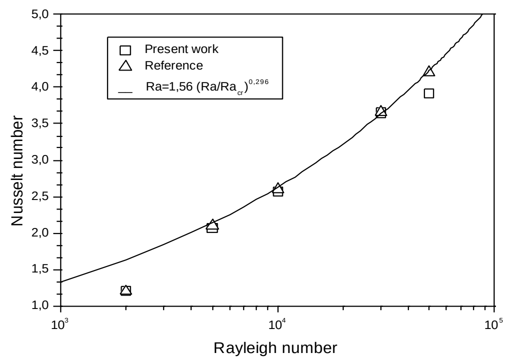

Fig. 4. Nusselt number vs. Rayleigh number for b/L=1.
Square: The current LB model; Triangles: Reference data of Ref. [24];
Line: Empirical power law $Nu = 1.56(Ra/Rac)^{0.296}$ [23].

B. Simulation

In this study, was used to predict the natural convection in a channel partially heated by studying the variation of the Rayleigh number. The geometry configuration is shown in Fig. 1. The portion heated in bottom wall is varied b/L ratio isvaried between 0,2 and 0,8 (i.e. 20%, 40%, 60%, 80% heated). The boundary conditions is similar at applied in validation case, except that between the heated portions we consider the adiabatic wall boundary in second order.

The heat transfer is described by the Nusselt number Nu, defined in (15), as the ratio between convective heat transport to the heat transport due to temperature conduction. For the computation of the Rayleigh number, the Nusselt number results is shown in Fig. 4.

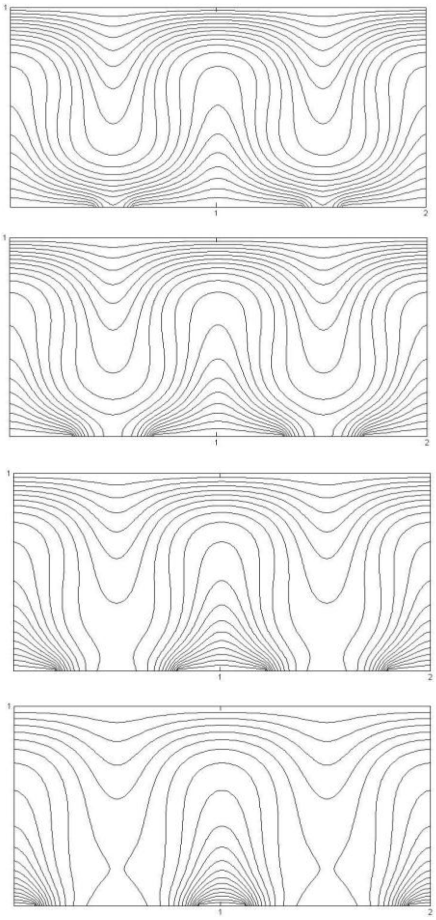

Fig. 5. Contour plot with iso-temperature line for Ra = 10^4.
The top to bottom b/L=0,8; 0,6; 0,4 and 0,2.

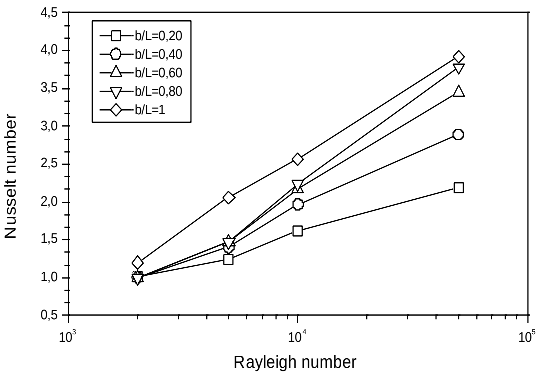

Fig. 7. Nusselt number vs. Rayleigh number for
b/L=1; 0.8; 0.60; 0.40; 0.20

The results obtained showed clearly that the phenomenon of transfer decrease according to the geometry heated. The isothermes and streamlines for the fluid flow is illustred in Fig. 2 and Fig. 3 what showed the appearance of four Benard rollers against rotative with height H (height of the channel) and width L/2 for all configurations.

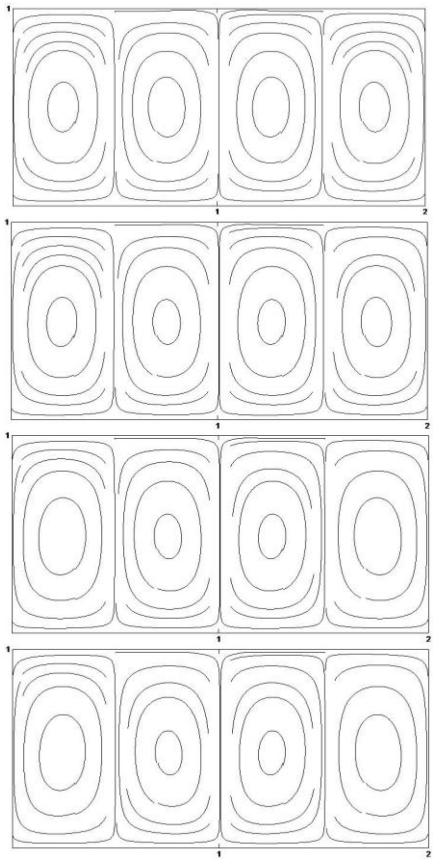

Fig. 6. Contour plot with streamlines for Ra=10^4.
The top to bottom b/L=0,8; 0,6; 0,4 and 0,2.

V. Conclusion

We have presented a lattice Boltzmann thermal model for convection heat transfer in an infinite horizontal channel partially heated in bottom wall. In this model, the temperature field is modeled by a new lattice Boltzmann equation, while the velocity field is simulated by the lattice Boltzmann isothermal model for flows. The present model has all the advantages, including good numerical stability and the ability to handle convection heat transfer problems. The numerical results of the used problem for validation in a channel totally heated by bottom wall, have demonstrated the accuracy and reliability of the used LBM, and the good agreement between the results obtained and the results in the literature.

This numerical model was used to predict and to study the natural convection in a channel partially heated by studying the variation of the Rayleigh number and the heated geometrical portion.

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