Modeling and Error Analysis of the Parallelism Measurement for Linear Rolling Guide Pair

Yi Liang, Yulin Wang, Aihua Yin, Li Zu, and Dan Xu

***Abstract*—The parallelism of a linear rolling guide pair directly affects the machining precision of machine tools. Therefore, it's necessary and meaningful to find the measurement method to test them. A test method of parallelism of rolling linear guide pairs based on non-contact laser displacement sensors has been put forward. The paper derives an analytical testing algorithm of measurements of five laser displacement sensors to obtain the parallelism and their error analysis of linear rolling guide pairs. The parallelism of linear guide pairs includes five movement precisions which are horizontal parallelism, vertical parallelism, elevation angle, deflection angle and roll angle of a slider block relative to its guide way. The converting formulas from the laser sensors' measurement to the two parallelisms and three angles are derived based on the basic theory of three-points can determine a plane. Because the sensors are installed with inevitably errors, the deviation of two parallelisms and three angles caused by the errors are analyzed. Results show that two parallelisms and three rotation angles depend critically on the measuring length of five laser sensors. The paper provides an effective method for measuring the parallelism and makes progress of the automatic detection of a linear rolling guide pair.**

***Index Terms*—Error analysis, Linear roller guide, movement precision, parallelism.**

I. Introduction

The rolling linear guide pairs are widely used in automation equipments and high-end CNC tools as precision guide members, because of its high linear positioning accuracy. The parallelism, namely straightness, of guide pairs in translation motion is a significant index of accessing accuracy, since which directly affects their own service life and the machining precision of machine tools. Therefore, it's necessary and meaningful to test and evaluate the parallelism of guide pairs.

In recent years, a few of researchers and engineers had studied on how to test the precision of linear guide pairs. Liu and C. H. Liu used optical measurement technology to measure the guide straightness by an intuitive and reliable method [1], [2]. Based on relativity measure theory, Li Wei designed and manufactured three automatic measuring instruments by applying geometrical simulation and automatic measuring technique. The measure result indicated that the instrument advanced the measuring veracity and efficiency [3]. L. J. Zhu and C. D. Hu introduced some method of measuring guide straightness error [4], [5]. Y. Liu used two total stations cooperating with each other to measure the straightness of the guide rail [6]. P. Majda proposed a method of analytical examination of the influence of geometric errors in linear guide way on joint kinematic errors [7]. Yang Ye and Feng used a gantry with five non-contact probe to measure the accuracy of guide way, it has high detection accuracy and it is suitable for different specifications of guide rail [8]. J. Xiong introduced the dynamic measurement of friction and put forward two methods to measure and check motion accuracy [9]. J. L. Sun and M. H. Qian discussed the processing course in quest of linear error by GA [10]. Tao, Yin, Cai and Zhang introduced a system for testing linear motion guide. In the system, laser, optical fiber, displacement sensor, laser collimation technique, force sensor and computers were used to measure and process data. It solved the problem of measuring linear motion guide [11]. Zhong, Tao and Han studied the modeling and test method on the motion precision loss process under load condition [12]. Qing and Hong adopted position sensitive detector (PSD) and laser collimation technique to measure four-degree-of-freedom of linear guide rails, and BP neural network algorithm is used to correct the nonlinearity between actual and ideal output of PSD. The results showed that the four-degree of linear guide rails can be measured with high accuracy [13]. These scholars did many useful and practical works in testing the accuracy of linear guide pairs.

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II. The Test Method of Parallelism in Linear Rolling Guide Pairs

Movement precision refers to the difference between the real and the theoretical displacements of a specific point. According to Mechanical Industry Standards of China JB/T 175.4-2006, the acceptance specifications for the precision test of rolling linear guide includes two aspects, as shown in Fig. 1.

1) The parallelism between the slider movement and the guide bottom reference A, namely vertical parallelism.
2) The parallelism between the slider movement and the guide side reference B, namely horizontal parallelism.

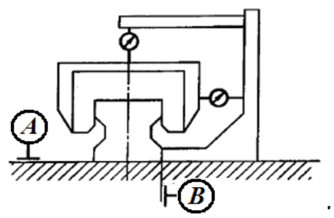

Fig. 1. The precision test of rolling linear guide.

When the guide works in normal, in theory, it could only be

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allowed to move in reciprocating linear motion along the rail direction. Actually, the movement of rolling elements in linear guide pairs is complex. The movement precision is affected by many factors, the manufacture precision of the parts, installation accuracy, preload force , frictional force etc. Therefore, in addition to the moving direction, there are five kind of errors in linear guide pairs, which are the parallelism accuracy (horizontal and vertical) and the accuracy of rotation around the three axes (elevation angle, deflection angle, roll angle). Though there are no standard of the accuracy of rotation in JB/T 175.4-2006, we insist test the angles to research linear guide slider's position and orientation because they would influence the machining accuracy.

A coordinate system is established in Fig. 2. Point O is the ideal position of the block on the linear guide, O' the actual position. So the translation displacement along to the X and Y direction refer to the horizontal and vertical parallelism of the linear guide pair, respectively. The angles of rotation around three axes, X, Y and Z, are the pitch angle θ_x , deflection angle θ_y and roll angle θ_z , respectively.

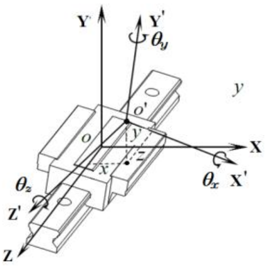

Fig. 2. The coordinate system of a rolling linear guide pair.

In Fig. 3, we designed an instrumentation to measure the horizontal and vertical parallelism and three angle errors of the linear guide pair.

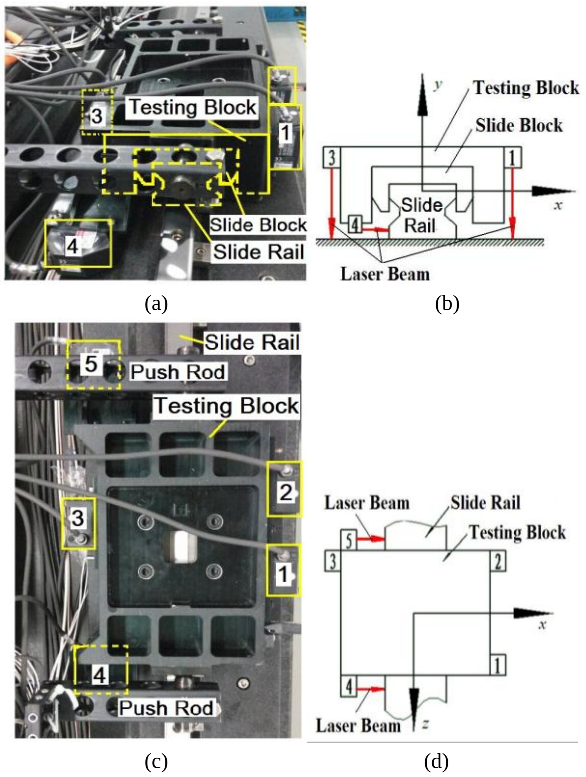

Fig. 3. Structure of test system.

Fig. 3(a) and Fig. 3(b) show the front view of measuring device, a practical photograph and schematic diagram respectively. Fig. 3(c) and Fig. 3(d) are the top view of them. Two push rods in Fig. 3(c) may push the testing block and slide block move along the rail back and forth, when testing will be carried out. In the Fig. 3, 1 to 5, five non-contact laser displacement sensors, are installed on the testing block, which is mounted on the slide block to be measured. The laser light of the sensors 1, 2 and 3 beam down on the granitic surface which is a reference plane. The laser light of the sensors 3 and 4 beam on one side of guide. When the slider is driven to move along the guide rail, the laser displacement sensors 1-5 would test the displacement variation (x_1, x_2, x_3, x_4, x_5,). Subsequently, the measured result would be saved and analyzed by the dynamic measurement software to calculate parallelisms and angle errors.

III. The Derivation of Parallelism of Rolling Linear Guide Pairs

A. *The Angle of Rotation*

Because the five laser sensors are perpendicular to their relevant test reference surfaces, the angles of slider block can be calculated by the measurement of sensors. So the pitch angle θ_x of the slider around the X direction is obtained by the measurement of sensors 1 and 2. The deflection angle θ_y around the Y direction can be calculated by the measurement results of sensors 4 and 5. The roll angle θ_z around the Z direction can be measured with sensors 2 and 3. The geometrical relationship and measuring principle are shown in Fig. 4.

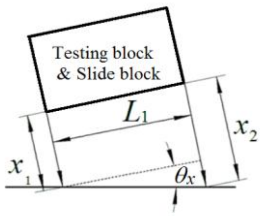

Fig. 4. Measuring principle of slider angle.

By the geometric relationships, the formulas of rotation are

$$\begin{cases} \theta_x = \arctan\left(\dfrac{x_1 - x_2}{L_1}\right) \\ \theta_y = \arctan\left(\dfrac{x_2 - x_3}{L_2}\right) \\ \theta_z = \arctan\left(\dfrac{x_4 - x_5}{L_3}\right) \end{cases} \tag{1}$$

where

θ_x, θ_y, θ_z-The angles of slider around X, Y, Z axis;
x_1, x_2, x_3, x_4, x_5-The measurements by five laser sensors;
L_1-The distance between the light spot of sensors 1 and 2;
L_2-The distance between the light spot of sensors 2 and 3;
L_3-The distance between the light spot of sensors 4 and 5.

B. The Horizontal Parallelism along the X Direction

According to Mechanical Industry Standards of China JB/T 175.4-2006, the horizontal parallelism x refers to variation of the distance between the center point of the side surface on the slide block and the side reference plane. However the laser distance sensors 4 and 5 on the front and back of the slide block, which is not the center point. So we need to find a method to convert the measurement of sensors to the horizontal of parallelism x on the center point.

The slider is regarded as a rigid body. Sensors 4 and 5 are symmetrically arranged, by calculating the distance of the slider side midpoint in the X direction. The horizontal parallelism x can be obtained by x_4, x_5 and h (see Fig. 5).

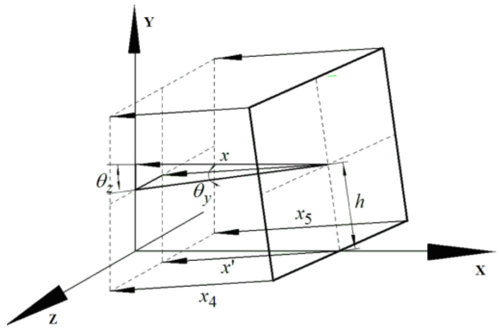

Fig. 5. The measuring principle of the horizontal parallelism.

Since a plane cannot be determined by two sensors, the compensation parameter h is added, which is the vertical distance from the center of slider side to the sensor mounting plane. The perpendicular distance x can be developed to

$$\begin{aligned} x &= x' \cos\theta_x \cos\theta_z - h\sin\theta_z \\ &= \frac{1}{2}(x_4 + x_5)\cos\theta_x \cos\theta_z - h\sin\theta_z \end{aligned} \tag{2}$$

C. The Vertical Parallelism along the Y Direction

Likewise, we need to derive the formula to convert the measurement of sensors 1, 2 and 3 to the vertical of parallelism y on the center point of the upper plane of the slider block. The three points of laser sensors can determine the upper plane, shown in Fig. 6. Since the sensors 1, 2 and sensors 2, 3 are installed symmetrically relative to the center plane of the slider, the vertical parallelism y of the slider surface center along the Y direction is equal to the moving distance in the Y direction of the upper plane center.

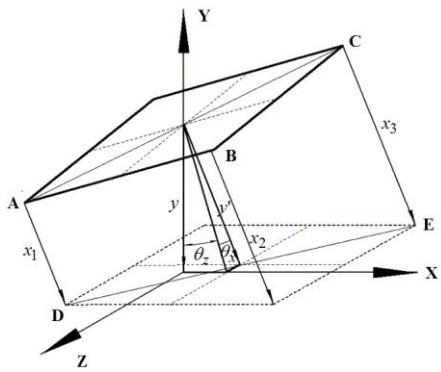

Fig. 6. The measuring principle of the vertical parallelism.

As shown in the Fig. 6, the trapezoid ADEC is formed in the space. Since the angle θ_y about the Y-axis does not affect the magnitude of y', it can be set to zero. So the perpendicular distance y can be developed to

$$y = y' \cos\theta_x \cos\theta = \frac{1}{2}(x_1 + x_3)\cos\theta_x \cos\theta_z \tag{3}$$

IV. The Effect of Installation Errors on the Parallelism

The sensors are mounted on the testing block with inevitably errors. Now we will analyze the deviation of the angle of rotation θ_x, θ_y, θ_z and linear parallelism x and y caused by the installation errors.

A. The Effect of Installation Errors on the Angle of Rotation

In Fig. 4, it's apparent that the angle of rotation θ_x depends on x_1, x_2 and L_1. If the actual length of these three parameters are not the exactly same with their ideal value, which is common in engineering field, the angle θ_x will be different from the value we expected. We denote the difference between ideal and actual value as Δx_1, Δx_2 and ΔL_1, respectively. The deviation of the angle of rotation θ_x cause by them is named denoted by $\Delta\theta_x$. We use the below formula to analyze the relationship of them.

$$\Delta\theta_x = \frac{\partial\theta_x}{\partial x_1}\Delta x_1 + \frac{\partial\theta_x}{\partial x_2}\Delta x_2 + \frac{\partial\theta_x}{\partial L_1}\Delta L_1 \tag{4}$$

In which, θ_x can be substituted by the formula (1).

In order to discovery the directive relation of ΔL_1 and $\Delta\theta_x$, shown in Fig. 7, the other factors Δx_1, Δx_2 will be fixed unchanged. Fig. 7 shows that the deviation of $\Delta\theta_x$ is positively correlated to ΔL_1. Furthermore, if the length L_1 gets smaller, namely, the sensors 1 and 2 are mounted closer, ΔL_1 has more influence on $\Delta\theta_x$. But even so, the deviation of $\Delta\theta_x$ is tiny to 0.00025 °when ΔL_1 has a large error of 5mm. So the error of ΔL_1 would not be a critical factor to $\Delta\theta_x$.

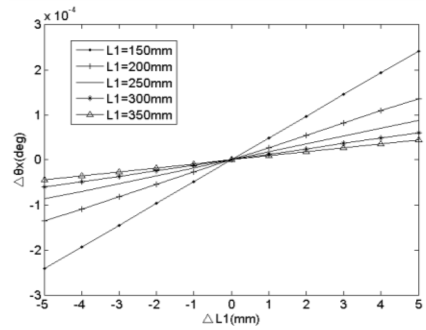

Fig. 7. The effect of error ΔL_1 on the angle $\Delta\theta_x$.

Similarly, when we analyze the influence of Δx_1, Δx_2 and ΔL_1 will be unchanged. Furthermore, because the reference measurement length x_1 of laser sensor 1 is 20mm and the measuring range of laser sensor 1 is 2mm, the actual measurement length x_1 would vary from 18mm to 22mm. When it changes in this range, the variation curves of $\Delta\theta_x$ are very close to each other and no obvious difference. So, the other curves except x_1=20mm are left out and are not shown in Fig. 8. It shows that the deviation of $\Delta\theta_x$ is positively correlated to Δx_1. Because the measuring range of laser sensor 1 is 2mm, the installation error Δx_1 is selected to -2mm to 2mm. Fig. 8 shows that the Δx_1 has a strong influence on $\Delta\theta_x$. When the error Δx_1 is 2mm, $\Delta\theta_x$ would have a relative huge deviation of 0.45°. So accuracy of the length x_1 is the key factor guaranteeing precision of the pitch angel θ_x.

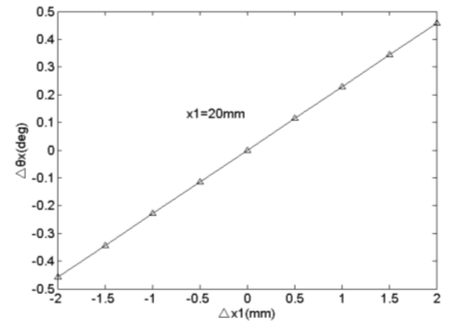

Fig. 8. The effect of error Δx_1 on the angle $\Delta\theta_x$.

In formula (1), we know that x_2 plays the same role with that of x_1 in determining the pitch angel θ_x. We have no use for drawing the variation curves of $\Delta\theta_x$ and Δx_2.

From formula (1), it can be seen that θ_y, θ_z are almost the same to θ_x. So we needn't to analyze them repeatedly.

B. The Effect of Installation Errors on the Linear Parallelism

By comparing formulas (2) and (3), we find they are similar to each other. We choose a linear parallelism y to analyze. Observing formulas (1) and (3), we notice that parallelism y is a function of 7 variables. Similar to formula (4), we get the deviation of vertical parallelism Δy.

$$\Delta y = \frac{\partial y}{\partial x_1}\Delta x_1 + \frac{\partial y}{\partial x_2}\Delta x_2 + \frac{\partial y}{\partial x_3}\Delta x_3 + \frac{\partial y}{\partial x_4}\Delta x_4 + \frac{\partial y}{\partial x_5}\Delta x_5 + \frac{\partial y}{\partial L_1}\Delta L_1 + \frac{\partial y}{\partial L_3}\Delta L_3 \tag{5}$$

To simplify the question, three typical factors x_1, x_4, L_1 are chose to be analyzed. Fig. 9 shows that the deviation of Δy is positively correlated to Δx_1. When the installation error Δx_1 changes from -2mm to 2mm, Δy varies from -1mm to 1mm. It means that Δx_1 has a great effect on Δy. The accuracy of the length x_1 is a key factor guaranteeing precision of vertical parallelism y.

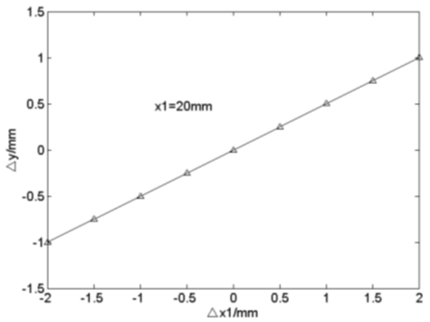

Fig. 9. The effect of error Δx_1 on the parallelism Δy.

In Fig. 10, the installation error of the laser sensor 4 is analyzed. Because the reference measurement length x_4 of laser sensor 4 is 50mm, the actual measurement length x_4 would vary from 48mm to 52mm. So, there are five curves in the figure. Fig. 10 shows that the deviation of Δy is proportional to Δx_4 when x_4 = 48, 49, 50mm and inversely proportional to Δx_4 when x_4 = 51, 52mm. However, if we focus on the absolute value of Δy, we find that $|\Delta y|$ is positively correlated to Δx_4. As for the curve of x_4 = 48 or 50mm, when the installation error Δx_4 changes from -5mm to 5mm, Δy varies from -0.0022mm to 0.0022mm. It means that Δx_4 has little effect on Δy. The accuracy of the length x_4 is not a critical factor to y.

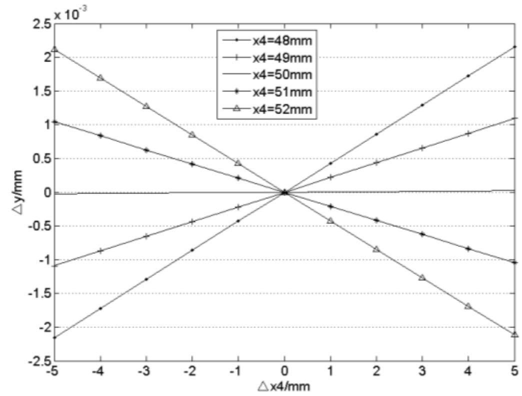

Fig. 10. The effect of error Δx_4 on the parallelism Δy.

In Fig. 11, the deviation of Δy is positively correlated to ΔL_3. Furthermore, if the length L_3 gets smaller, namely, the sensors 4 and 5 are mounted closer, ΔL_3 has more influence on Δy. But even so, the deviation of Δy is tiny, 6×10^{-6}μm when ΔL_3 has a large error of 5mm. So the error of ΔL_3 would not be a critical factor to Δy.

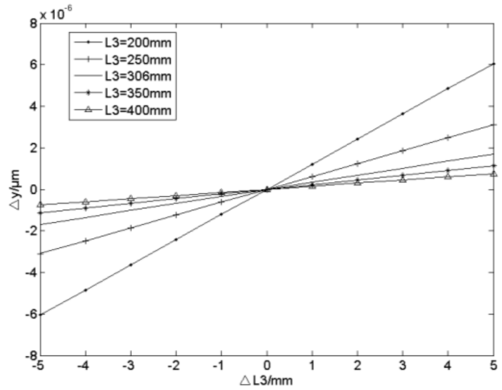

Fig. 11. The effect of error ΔL_3 on the parallelism Δy.

C. The Brief Summary of the Effects of Installation Errors on the Linear Parallelism

There are 9 variables in the instrumentation to measure the horizontal and vertical parallelism and three angle errors of the linear guide pair. In order to clarify their relationship and importance, Table I is listed.

Table I show that five measuring length x_1, x_2, x_3, x_4, x_5 are important to 5 precisions, the angle of rotation θ_x, θ_y, θ_z, and linear parallelism x. We need to make sure that the positions of sensors would keep fixed and unchanged while testing.

TABLE I: THE RELATIONSHIP AND IMPORTANCE OF 7 VARIABLES TO 5 PRECISIONS

	θ_x	θ_y	θ_z	x	y
x_1	●			○	●
x_2	●	●		○	○
x_3		●			●
x_4			●	●	○
x_5			●	●	○
L_1	○			○	○
L_2		○			
L_3			○	○	○
h				●	
●: interrelated & critical ○: interrelated					

V. CONCLUSION

It is necessary and meaningful to test and evaluate the parallelism of guide pairs. A test instrumentation and method by using five non-contact laser displacement sensors have been put forward to measure the horizontal, vertical parallelism, x, y, and three rotation angles θ_x, θ_y, θ_z of the linear guide pair. The concerting formulas from the laser sensors' measurement to the parallelism of rolling linear guide pairs are derived. Because all the sensors are mounted on the testing block with some inevitably errors, the deviation of the angle of rotation θ_x, θ_y, θ_z and linear parallelism x and y caused by the errors are analyzed. Results show that two parallelism, x, y and three rotation angles θ_x, θ_y, θ_z, depend critically on five measuring length x_1, x_2, x_3, x_4, x_5. The error of them would cause great measurement error of two parallelism and three rotation angles.

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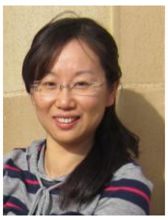

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