Complex Research of Dynamics of Control Systems with Fuzzy P–Controller

Nina A. Kazachek, Valery M. Lokhin, and Vladimir A. Ryabcov

***Abstract*—In this paper, the method of complex research of systems with fuzzy P-controller, based on the criteria of absolute stability of VM Popov and the harmonic balance method is presented. A rationality of using of these methods in such systems is explained, taking into account the features of nonlinear transformations in fuzzy P-controller and parameters affecting the character of the nonlinear. By varying the parameters of the nonlinearity, taking into account the influence of the parameters of the nonlinear characteristics of the fuzzy P-controller on the dynamics of the system, the authors have developed a generalized technique of forming a non-linear transformation in the fuzzy P-controller And the examples and simulation results are shown to verify the effectiveness of the proposed methods.**

***Index Terms*— Stability, PID controller, fuzzy P-controller, the stability criteria of VM Popov, the harmonic balance method.**

I. Introduction

In recent years, fuzzy logic controllers (FC) based on intellectual technology of fuzzy inference (FI) occupy more wide niche in practice of automatic control system (ACS) creating [1], [2]. As shown, in particular, in [1]-[4], application of such controllers is very perspective, because, it improves the quality indexes of the control process (primarily, such as response time and overshoot), reduces the influence of external disturbances, and increases the steady-state accuracy.

However, the theoretical foundation, which provides the problems solving of analysis and synthesis of such controllers, is increasingly behind from the general engineering practice.

In assessing the complexity of the issue, it should be emphasized that even the adjusting of the classic PID - controller (where it is necessary to vary all three parameters) is associated with certain difficulties. In case of fuzzy PID-controller setting, the numbers of such parameters are increasing in several times more.

It should be noted that the PID - controller (as a classical linear and fuzzy) is a versatile construction and for many industrial facilities is redundant. Therefore, for this class of objects it is possible to use a simplified modification of the controller, precisely a fuzzy P- controller (FPC). Consider the ACS with the FPC according to the theory non-linear systems, considering features of nonlinear transformations, implemented in such a controller

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II. The Description of the Object of Study

Block diagram of the ACS with the FPC is shown in Fig. 1 As shown by studies [3], [4], all transformation in the fuzzy controllers is essentially non-linear. This explains the improvement of the quality of control ACS with fuzzy controllers: the value of the gain of the FPC is changing in the process of processing the reference variable (Fig. 2). Parameters of the nonlinear characteristic of the FPC depend on:

- The dimension of the term - the sets of linguistic variables;
- The shape and relative position of the membership functions of individual terms;
- The priority of the relationship of input and output terms.

As can be seen, there are many FI parameters, influencing to the character of the nonlinear transformation of FPC. In writing this paper, the authors believe that the reader is familiar with the basic concepts of fuzzy logic or can be found in [1]-[4].

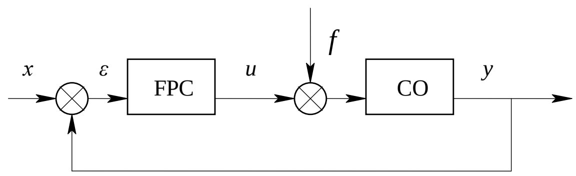

Fig. 1. Block diagram of the ACS with fuzzy P-controller (FPC); CO-control object, x – reference variable, f – external disturbances.

III. The Techniques of Forming a Nonlinear Transformation in the FPC

Generalized techniques of forming a non-linear transformations in the FPC (Fig. 2a), can be explained by Fig. 3. Where the initial characteristic 1 corresponds to a uniform distribution of seven triangular membership functions of input and output variables, rule base is shown in Fig. 4. To reduce the inclination of the nonlinearity at the origin of coordinates and to establishment of the inflection point, the input and output membership functions, which are responsible for the central portion (in this case, this is the input function «D» and output function "4"), are stretched. As a result of changes, the required curve "2" is formed (the dashed curve "3" approximates the obtained characteristic by

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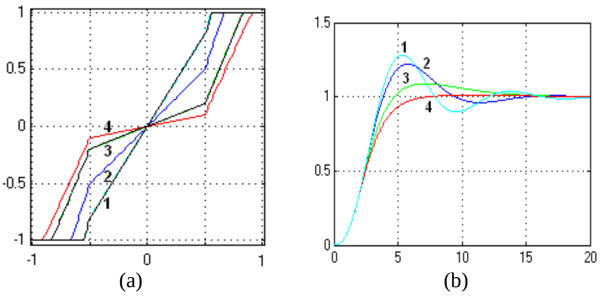
two straight sections). The rule base remains unchanged.

IV. The Criteria of Absolute Stability of VM Popov

As shown in Fig. 2, the transformation $U(\varepsilon)$ in the RPC is non-linear, therefore, a research problem of system dynamics with this type of non-linear characteristic arises. It is obvious that for research of the absolute stability of ACS with the FPC it possible apply the analog of the VM Popov criterion [5].



(a) (b)

Fig. 2. The influence of the nonlinear characteristic of the FPC (a) on the quality of the transition process (b)

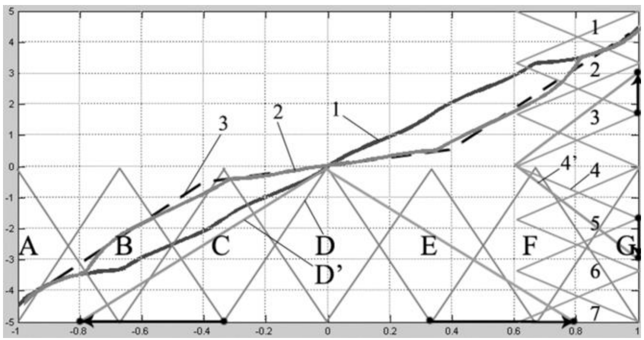


Fig. 3. Formation of nonlinear transformation in fuzzy controller.

1. If (input1 is mf1) then (output1 is mf1) (1)
2. If (input1 is mf2) then (output1 is mf2) (1)
3. If (input1 is mf3) then (output1 is mf3) (1)
4. If (input1 is mf4) then (output1 is mf4) (1)
5. If (input1 is mf5) then (output1 is mf5) (1)
6. If (input1 is mf6) then (output1 is mf6) (1)
7. If (input1 is mf7) then (output1 is mf7) (1)

Fig. 4. The rule base of fuzzy P controller.

According to this, the condition of stability of the equilibrium of such a system takes the form:

$$\mathrm{Re}[1 + jq\omega] \cdot W_{CO}(j\omega) + \frac{1}{k} > 0 \qquad (1)$$

where q - real number (positive or negative), W_{CO} frequency response of CO, k - the slope of the vector, limiting sector, within which are non-linear characteristics $U(\varepsilon)$. Here and further we assume that the control object has the order not lower than third and it is stable or neutral.

It is important to emphasize this fact, from two points of view:

- Firstly, many industrial objects can be represented by the model of this type;
- Secondly, the hodograph of such an object is convex (i.e., the point at phase -π is the most left); it has a great practical importance for the subsequent analysis of the dynamics of ACS with the FPC.

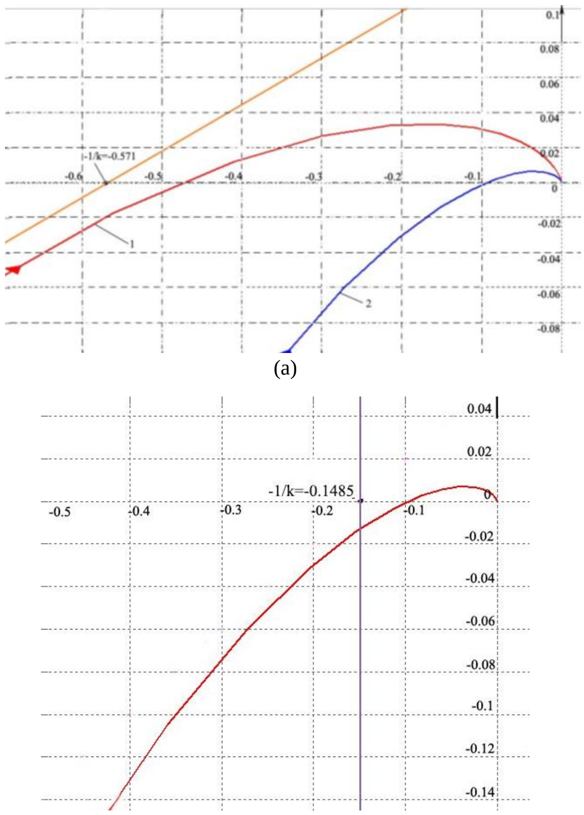
Graphical interpretation of the condition (1) for $W_{CO}(p) = K_{CO}[p(1+pT_1)(1+pT_2)]^{-1}$ is shown in Fig. 5a. Where $K_{CO} = 50$, $T_1 = 0{,}002$, $T_2 = 0{,}05$. As can be seen, the position of equilibrium of this system is stable; increasing K_{CO} by 5 times leads to loss of stability of the system (Fig. 6a).

There is the flex point in the nonlinear characteristic *U(ε)*, hence the derivative $dU(\varepsilon)/d\varepsilon$ will change. These facts point to the need to involve the criterion of absolute stability of processes, which for ACS with the FPC (Fig. 1, 2) can be written in the form:

$$\mathrm{Re}\cdot W_{CO}(j\omega) + \frac{1}{k_s} > 0 \qquad (2)$$
$$0 < \frac{\delta U(\varepsilon)}{\delta\varepsilon} < k_s$$

where k_s is the slope of the steepest part of the nonlinear characteristics *U(ε)*. Fig. 6b shows the transition process at x=0,2 for FPC with characteristic 2 in Fig. 2 and $W_{CO}(p) = 50[p(1+0{,}002p)(1+0{,}05p)]^{-1}$. As can be seen, the stability condition is not fulfilled, but the process is stable although has oscillations.

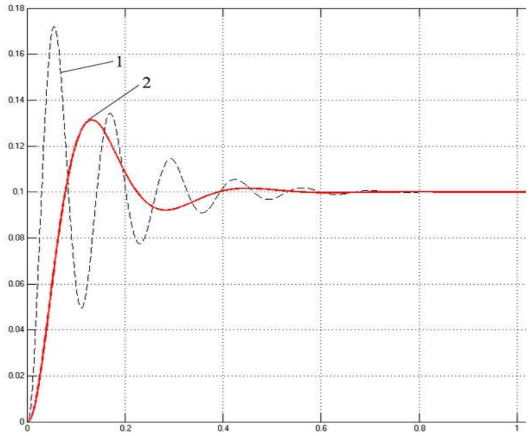
Comparison of conditions 1) and 2) shows that the criterion of the absolute stability of processes is more "rigid", that's why it will have to be used in analysis and synthesis parameters of FCP. At the same time it is important to emphasize that the conditions 1) and 2) are sufficient and consequently it is difficult to give a reliable estimate of the stability region.



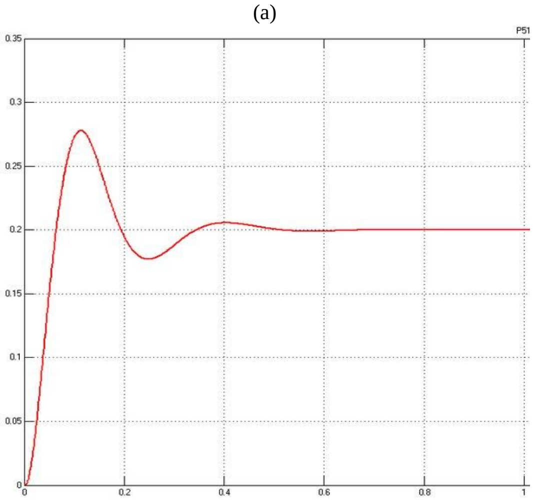
(a)

(b)

Fig. 5. Graph-analytical interpretation of the conditions for the absolute stability of ACS with fuzzy P-controller (a-equilibrium position, b - processes).



(a)



(b)

Fig. 6. Transients in ACS with the FPC.

V. The Method of Harmonic Balance

Since the criteria of absolute stability have sufficient character, we propose supplement the criterion of absolute stability with the method of harmonic balance. Because considered class of objects have the order not lower than third, then the filter hypothesis for these objects is executed with sufficient accuracy, and, correspondingly, the method of harmonic balance gives a reliable result.

Complex gain of the nonlinear element (similar to a presented in Fig. 2) can be written in the form (the intermediate results omitted in mind of simplicity):

$$W_{FPC}(A) = \frac{2}{\pi}[(k_1 - k_2)\arcsin\frac{a}{A} + k_2 \arcsin\frac{b}{A} + \\ + \frac{2k_2 a + 2c - k_2 b}{A}\sqrt{1-\left(\frac{b}{A}\right)^2} - \frac{k_2 a + k_1 a}{A}\sqrt{1-\left(\frac{a}{A}\right)^2}], \tag{3}$$

$$k_1 = \frac{d}{a}, k_2 = \frac{c-d}{b-a}$$

where A is input signal amplitude; k_1 and k_2 are slope of curve *1*-th and *2*-th part of the nonlinear characteristics $U(\varepsilon)$; (a,d) and (b,c) are coordinates of flex point of the nonlinear characteristic $U(\varepsilon)$.

Characteristic $-W_{FPC}{}^{-1}(A)$ built in Fig. 7 for graph-analytical solutions of the harmonic balance.

The solution of equation of harmonic balance is reduced to the definition of the points of intersection of the characteristics $-W_{FPC}{}^{-1}(A)$ and $W_{CO}(j\omega)$. Fig. 7 shows the built hodograph $W_{CO}(j\omega)$ for the same object that was considered above.

As can be seen, there are no oscillations in the system. However, the values that define the boundary of the absence of oscillations, closer to the values that define the range of absolute stability of processes, but not to the values that define the range of absolute stability of the equilibrium (Fig. 5).

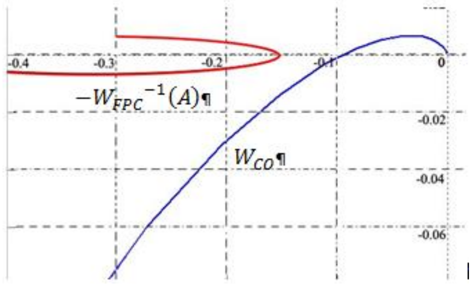


Fig. 7. Graph-analytical solution of the harmonic balance of ACS for FPC.

Availability of suitable graph-analytical techniques based on the relations (1) - (3) allows comprehensive study the dynamics of ACS with the FPC from the position of the two methods. Using the advantages of each method, we get the generalized picture of dynamics, which is more reliable in comparison with that gives each method alone.

At the same time, necessary to consider that using absolute stability criterion, we obtain an indirect assessment of quality - the stability index, and using the harmonic balance - an indirect estimate of oscillation index, by using known methods, given, for example in [6] - [9]. Thus, researching the region of stability of ACS with FPC, we can analyze the response time and overshoot at the same time.

VI. Conclusion

This paper discusses the complex application of the method of absolute stability of nonlinear systems and the method of studying periodic oscillations in combination with indirect estimates of the quality of transients.

And based on these methods, the convenient for the general engineering practice, the graph-analytical method of analysis and synthesis of ACS with fuzzy P-controller is proposed

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