Numerical Simulation of Conjugate Free Convection in a Vertical Cylinder Having Porous Layer

Mikhail A. Sheremet

Abstract—A numerical study of the conjugate natural convection heat transfer in a vertical cylindrical cavity having a vertical porous layer has been carried out. The vertical cylinder has a heat-conducting solid shell of finite thickness and conductivity. Convective heat exchange with an environment is modeled at the top and side walls while the bottom wall is adiabatic. The mathematical model has been formulated in dimensionless stream function, vorticity and temperature taking into account the Darcy-Boussinesq approximation for the porous layer. The boundary-value problem has been solved numerically on the basis of a second-order accurate finite difference method. Efforts have been focused on the effects of three types of influential factors such as the porous layer thickness, Biot number and dimensionless time on the fluid flow and heat transfer parameters. It has been found that an increase in the Biot number leads to an essential intensification of convective heat transfer at initial time level.

Index Terms—Free convection, vertical cylinder, heat-conducting solid shell, Darcy porous layer.

I. INTRODUCTION

Free convection in porous enclosures has received considerable attention in recent years because of its relation to the thermal performance of many engineering installations [1]-[5]. Natural convection heat transfer in domains containing pure fluid and porous medium is a fundamental transport mechanism encountered in a wide range of engineering, geophysics, and scientific applications, such as packed bed solar energy storage, fibrous and granular insulation systems, water reservoirs, post-accident cooling of nuclear reactors, etc. [1]-[5].

Laminar natural convection in partially porous enclosures has been investigated numerically and experimentally [1]-[9]. Singh and Thorpe [6] have numerically analyzed the Darcy, Brinkman and Brinkman-Forchheimer models of natural convection in a differentially heated square enclosure containing simultaneously a pure fluid and a horizontal porous layer. It has been found that for moderate to small Darcy numbers the results obtained on the basis of the Darcy model with the Beavers-Joseph empirical condition at the fluid-porous interface are similar to the results obtained by the Brinkman and Brinkman-Forchheimer models. Sheremet and Trifonova [7], [8] have numerically studied the Darcy and Brinkman models of conjugate natural convection in a

vertical cylinder containing horizontal pure fluid and porous layers. It has been shown that at small values of the thermal conductivity ratio the linear Darcy model is valid for the conjugate natural convection problems. An increase in the thermal conductivity ratio leads to more essential quantitative differences between the results obtained on the basis of the Darcy and Brinkman-extended Darcy models. Liu et al. [9] have experimentally investigated free convection in a two-dimensional cavity partially filled with a vertical porous layer. It was found that the value of average slip coefficient would vary from 0.307 to 2.53 for the range of Rayleigh number from 2.806×104 to 1.053×106. Mharzi et al. [10] have numerically analyzed laminar thermosolutal natural convection inside a square cavity containing simultaneously a binary fluid and a saturated vertical porous layer. It has been found that the thermal and solutal exchange are sensitive to the Rayleigh and Darcy numbers, so the increase in Ra leads to an intensification of convective heat transfer inside the fluid layer while the convection heat transfer inside the porous layer is enhanced with the increase in the Darcy number. Singh et al. [11] have analyzed the transient natural convection in a vertical channel partially filled with a porous medium. The authors have used Forchheimer-Brinkman extended Darcy model for simulation of the momentum transfer within the porous medium. Using the perturbation technique the authors have shown that the increase in the fluid region size leads to the increase in the porous region velocity. It has been found also that the velocity is more sensitive for higher values of thermal conductivity ratio in comparison to lower values of thermal conductivity ratio in fluid region. Lai and Kulacki [12] have studied steady-state free convection in a differentially heated square cavity filled with a two-layer porous system using the Darcy-Boussinesq approximation. It should be noted that the authors have used special boundary conditions at porous-porous interface [13], [14]. The authors have found that the heat transfer begins as a conduction heat transfer in the less permeable sublayer and as a thermal convection in the layer with higher permeability. At the same time convective heat transfer in the layer of higher permeability begins to penetrate into the less permeable laver with an increase in the Rayleigh number. Double diffusive natural convection in a vertical cavity containing a layer of a binary fluid and a porous layer has been studied by Gobin et al. [15]. The authors have used the macroscopic conservation equations with both Darcy-Brinkman model in the porous layer and Navier-Stokes model in the binary fluid layer. Using the standard finite volume procedure the authors have found that the presence of the porous layer has a strong influence on the heat and mass transfer. Baytas et al. [16] have investigated double diffusive natural convection

Manuscript received May 26, 2015; revised September 12, 2015. This work was conducted as a government task of the Ministry of Education and Science of the Russian Federation, Project Number 13.1919.2014/K.

Mikhail A. Sheremet is with the Department of Theoretical Mechanics, Tomsk State University, Tomsk, 634050 Russia (e-mail: Michael-sher@yandex.ru).

between a saturated porous layer and an overlying fluid layer in a square cavity using Forchheimer-Brinkman extended Darcy model for simulation of the momentum transfer within the porous medium. It has been found that if the fluid-porous interface is not horizontal and contains a step change in height, the convective heat and mass transfer is changed essentially. Bagchi and Kulacki using [17] Forchheimer-Brinkman extended Darcy model have analyzed natural convection in a fluid-superposed porous layer heated locally from below. It has been found that the average heat transfer coefficient over the heat source surface increases with a decrease in the heat source length and decreases with a decrease in the Darcy number. Further on Bagchi and Kulacki [18] experimentally have analyzed the above-mentioned problem. Experiments have been carried out for a glass-water system. The obtained results have shown a formation of a plume-like flow with a single pair of circulating cells and convective motion inside the porous layer. Arpino et al. [19] numerically have analyzed axial flow convection in cylindrical domains completely or partially filled with a fluid saturated porous medium. The authors have proposed a stable and effective axisymmetric Artificial Compressibility Characteristic Based Split scheme for the considered physical problem. Merrikh and Mohamad [20] have studied natural convection in a square cavity filled with two vertical layers of porous medium using the Marker and Cell technique for incompressible flow [21]. It has been found that there is an essential difference between linear Darcy model and the general model predictions and in several cases it is necessary to use no-slip boundary conditions both at the cavity walls and at the interface between the porous media.

From the above literature survey it is evident that conjugate natural convection within cavity containing vertical fluid layer and porous layer in conditions of convective heat exchange with an environment has not been studied in detail. The main objective of the present study is to numerically analyze the unsteady natural convection in a vertical cylindrical cavity partially filled with a porous medium having heat conducting solid walls. To our best of knowledge this problem has not been considered before, so that the reported results are new and original.

II. GOVERNING EQUATIONS AND NUMERICAL METHOD

The domain of interest consists of a central vertical pure fluid layer and vertical porous medium layer of thickness dlocated close to the walls in a vertical cylindrical cavity of height D and radius L bounded by solid heat-conducting shell of finite thickness l is shown in Fig. 1.

The direction of gravity is along the cylinder axis (*z*-axis) while the *r*-axis is taken in the radial direction. The fluid-porous interface is assumed to be permeable so that the fluid can penetrate into the porous layer. The external surface of the bottom wall (z = 0) is considered to be adiabatic. The convective heat exchange with an environment is modeled on other borders like r = L+l and z = D+2l. The ambient temperature T^e is assumed to be constant and less than an initial temperature T_0 of the domain of interest. Therefore

during this investigation it is possible to analyze an effect of porous medium on the heat insulation of the domain of interest. All internal surfaces of the solid shell are assumed to be impermeable. For limiting the number of independent parameters, all physical properties of the fluid are supposed constant with temperature except for the density in the buoyancy term in the momentum equations where the Boussinesq approximation is valid. The porous medium using the Darcy's law is considered homogeneous and isotropic and is saturated with a fluid which is in local thermodynamic equilibrium with the solid matrix. In addition, the flow and heat transfer are transient, laminar and two-dimensional.



Fig. 1. Physical model and coordinate system (1-solid walls; 2-pure fluid layer; 3-porous medium layer).

The governing conservation equations for the fluid and porous layers and also for the solid shell have been formulated in dimensionless form using stream function, vorticity and temperature variables [7], [8]:

• for the pure fluid layer:

$$\frac{\partial^2 \Psi}{\partial R^2} - \frac{1}{R} \frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial Z^2} = -R\Omega, \tag{1}$$

$$\frac{\partial\Omega}{\partial\tau} + \frac{\partial(U\Omega)}{\partial R} + \frac{\partial(V\Omega)}{\partial Z} = \sqrt{\frac{\Pr}{Ra}} \times$$

$$\times \left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Omega}{\partial R}\right) + \frac{\partial^{2}\Omega}{\partial Z^{2}} - \frac{\Omega}{R^{2}}\right] + \frac{\partial\Theta}{\partial R},$$

$$\frac{\partial\Theta}{\partial Q} + \frac{\partial(U\Theta)}{\partial Q} + \frac{\partial(V\Theta)}{\partial Q} = -\frac{1}{R^{2}} \times$$
(2)

$$\times \left[\frac{1}{\partial \tau} + \frac{\partial (\nabla T)}{\partial R} + \frac{\partial (\nabla T)}{\partial Z} + \frac{\partial (\nabla T)}{\partial Z} - \frac{1}{\sqrt{\text{Ra} \cdot \text{Pr}}} \right] \times (3)$$

$$\times \left[\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial \Theta}{\partial R}\right) + \frac{\partial^2 \Theta}{\partial Z^2} - \frac{U \Theta}{R}\right]$$

• for the porous medium layer:

$$\frac{\partial^2 \Psi}{\partial R^2} - \frac{1}{R} \frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial Z^2} + \mathrm{Da} \sqrt{\frac{\mathrm{Ra}}{\mathrm{Pr}}} \cdot R \frac{\partial \Theta}{\partial R} = 0, \qquad (4)$$

$$\frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial R} + V \frac{\partial \Theta}{\partial Z} = \frac{\alpha_{3,2}}{\sqrt{\text{Ra} \cdot \text{Pr}}} \times \left[\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial \Theta}{\partial R} \right) + \frac{\partial^2 \Theta}{\partial Z^2} \right],$$
(5)

• for the heat-conducting solid shell:

$$\frac{\partial \Theta}{\partial \tau} = \frac{\alpha_{1,2}}{\sqrt{\mathrm{Ra} \cdot \mathrm{Pr}}} \left[\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial \Theta}{\partial R} \right) + \frac{\partial^2 \Theta}{\partial Z^2} \right]. \tag{6}$$

Here *R*, *Z* are the dimensionless cylindrical coordinates; Ψ is the dimensionless stream function; Ω is the dimensionless vorticity; Θ is the dimensionless temperature; τ is the dimensionless time; *U*, *V* are the dimensionless velocity component in *R*- and *Z*-direction, respectively; Ra is the Rayleigh number; Pr is the Prandtl number; Da is the Darcy number; $\alpha_{i,j} = \alpha_i / \alpha_j$ is the thermal diffusivity ratio.

The boundary conditions for the governing equations Eq. 1–Eq. 6 are as follows [7], [8]:

- at R = 1 + l/L and Z = D/L + 2l/L the conditions considering convective heat exchange with an environment are realized $\partial \Theta / \partial \overline{n} = -Bi\Theta$;
- adiabatic condition is set on the boundary Z = 0 $\partial \Theta / \partial Z = 0$;
- at axis of symmetry R = 0: $\Psi = \Omega = \partial \Theta / \partial R = 0$;
- at internal solid-porous interface R = 1 and l/L < Z < l/L + D/L:

$$\Psi = 0, \ \frac{\partial \Psi}{\partial R} = 0, \ \Theta_1 = \Theta_3, \ k_{1,3} \frac{\partial \Theta_1}{\partial R} = \frac{\partial \Theta_3}{\partial R};$$

• at internal solid-fluid interfaces Z = l/L and Z = D/L + l/L for 0 < R < 1 - d/L:

$$\Psi = 0, \ \frac{\partial \Psi}{\partial Z} = 0, \ \Omega = -\frac{1}{R} \frac{\partial^2 \Psi}{\partial Z^2}, \ \Theta_1 = \Theta_2, \ k_{1,2} \frac{\partial \Theta_1}{\partial Z} = \frac{\partial \Theta_2}{\partial Z};$$

• at internal solid-porous interfaces Z = l/L and Z = D/L + l/L for 1 - d/L < R < 1:

$$\Psi = 0, \ \frac{\partial \Psi}{\partial Z} = 0, \ \Theta_1 = \Theta_3, \ k_{1,3} \frac{\partial \Theta_1}{\partial Z} = \frac{\partial \Theta_3}{\partial Z}$$

• at internal fluid-porous interface R = 1 - d/L:

$$\begin{cases} \Psi_2 = \Psi_3, \\ \frac{\partial^2 \Psi_2}{\partial R^2} = \frac{\kappa}{\sqrt{Da}} \left(\frac{\partial \Psi_2}{\partial R} - \frac{\partial \Psi_3}{\partial R} \right), \\ k_{2,3} \frac{\partial \Theta_2}{\partial R} = \frac{\partial \Theta_3}{\partial R}. \end{cases}$$

Here Bi is the Biot number; κ is the coefficient in the Beavers-Joseph condition; $k_{i,j} = k_i/k_j$ is the thermal conductivity ratio.

The considered conditions at internal fluid-porous interface for the stream function are the conditions of Beavers and Joseph [22]. Neale and Nader [23] presented an analysis for the flow in a channel having fluid and porous zones and found that the Darcy model with such conditions gives the same results as that obtained by using the Brinkman model by considering continuity of the velocity and shear stress at the fluid-porous interface. In the initial time point we $\Psi(R,Z,0)=0,$ used the following conditions $\Omega(R,Z,0) = 0$, $\Theta(R,Z,0) = 1.0$. Therefore we can analyze an effect of the ambient cooling on the porous layer system bounded by solid walls of finite thickness and conductivity.

The partial differential equations Eq. 1-Eq. 6 with corresponding initial and boundary conditions were solved by finite difference method [7], [8], [24], [25] using the uniform grid. For an approximation of the convective terms we used the monotonic Samarskii scheme of the second order,

allowing considering a sign of velocity and for an approximation of the diffusion terms we used the central differences. The parabolic equations were solved on the basis of Samarskii locally one-dimensional scheme. The discretised equations were solved by Thomas algorithm. The equations for the stream function (Eq. 1 and Eq. 4) were discretised by means of five-point difference scheme on the basis of central differences for the second derivatives. The obtained difference equations were solved by the successive over relaxation method. Optimum value of the relaxation parameter was chosen on the basis of computing experiments.



Fig. 2. Comparison of streamlines Ψ and isotherms Θ at Da = 10–5, Ra = 105: numerical results of Singh and Thorpe [6] (a), present study (b).



Fig. 3. Comparison of streamlines Ψ and isotherms Θ at Da = 10⁻⁵, Ra = 10⁶: numerical results of Singh and Thorpe [6] (*a*), present study (*b*).

The accuracy of the numerical code developed by the author was checked by preparing the benchmark solutions both for non-conjugate and conjugate natural convection in porous and pure media [7], [8]. In the case of non-conjugate problem we have analyzed laminar natural convection in a square cavity containing a fluid layer overlying a porous layer saturated with the same fluid [6]. Fig. 2 and Fig. 3 show a good agreement between the obtained streamlines and isotherms at different Rayleigh numbers and the results by [6]. Results on the basis of Brinkman model is (-----) and on the basis of Darcy model is (-----) in Fig. 2(a) and Fig. 3(a).

III. RESULTS AND DISCUSSION

Numerical investigation of the boundary value problem has been carried out at following values of dimensionless complexes: Ra = 10^6 ; Da = 10^{-5} ; Pr = 0.71; D/L = 1; l/L = 0.1; $k_{1,3} = 20$; $1 \le \text{Bi} \le 10$; 0 < d/L < 1; $0 \le \tau \le 1000$.

Particular efforts have been focused on the effects of three types of influential factors such as the Biot number, the thickness of the porous layer, and the dimensionless time on the fluid flow and heat transfer.

Fig. 4 presents streamlines and isotherms at Bi = 1, $\tau = 1000$ and different values of the porous medium layer thickness.



It should be noted that the considered problem is essentially transient due to an effect of convective cooling of the domain of interest from outside. Therefore stream function and temperature fields are presented at the end of the cooling process at $\tau = 1000$ (Fig. 5). Regardless of the porous medium layer thickness a toroidal vortex is formed in the cavity with convective core inside the pure fluid layer close to the internal fluid-porous interface owing to a difference between an initial temperature in the cavity and an ambient temperature in conditions of a convective cooling from the lateral surface and the top of cylindrical cavity. An increase in the porous medium layer thickness leads on the one hand to the displacement of the convective core to the cylindrical symmetry line due to reduction of the pure fluid layer thickness and on the other hand to weak drift of this core along the vertical direction. It is worth noting here that in the porous layer the heat conduction is a dominated heat transfer mechanism. Therefore an increase in the porous medium layer thickness also leads to an attenuation of the natural convective heat transfer. Thicker porous layer allows increasing essentially the cooling time for the internal cavity.

Effects of the dimensionless time, porous layer thickness and Biot number on the average Nusselt number $Nu_{avg} = \int_{l/L}^{1+l/L} \left| \frac{\partial \Theta}{\partial R} \right|_{R=1-d/L} dZ$ are presented in Fig. 5. It should be

noted that the average Nusselt number is a non-monotonic time function owing to cooling of the domain of interest with variation of the temperature differences inside the cavity. The latter characterizes an evolution in time of the convective flow intensity. An increase in d/L leads an increment in Nu_{avg} at $\tau > 600$. In case of $\tau < 600$ the dependence of Nu_{avg} = f(d/L) is essentially non-monotonic. Effect of the Biot number that defines the convective cooling intensity is presented in Fig. 5b. An increase in Bi leads to both a significant increase in the maximum value of Nu_{avg} and an intensive reduction of the average Nusselt number with dimensionless time.



Fig. 5. Variation of the average Nusselt number at the side solid-porous interface versus the dimensionless time and porous layer thickness (*a*) and the dimensionless time and Biot number (*b*).

IV. CONCLUSION

Numerical simulation of transient conjugate free convection in a vertical cylindrical cavity partially filled with a porous medium in conditions of convective cooling from an environment has been carried out. Distributions of streamlines and isotherms in a wide range of key parameters have been obtained. It has been found that an increase in the porous layer thickness leads to an essential reduction of the cooling intensity. It has been shown also that an increment in Bi leads to both a significant increase in the maximum value of the average Nusselt number at internal porous–solid interface and an intensive reduction of this integral parameter with dimensionless time.

REFERENCES

- [1] A. Bejan, *Convection Heat Transfer*, New Jersey: John Wiley and Sons, 2004.
- [2] D. A. Nield and A. Bejan, *Convection in Porous Media*, New York: Springer, 2013.
- [3] A. Bagchi and F. A. Kulacki, *Natural Convection in Superposed Fluid-Porous Layers*, New York: Springer, 2014.
- [4] I. Pop and D. B. Ingham, Convective Heat Transfer: Mathematical and Computational Modeling of Viscous Fluids and Porous Media, Oxford: Pergamon, 2001.
- [5] D. B. Ingham and I. Pop, *Transport Phenomena in Porous Media III*, Oxford: Elsevier, 2005.
- [6] A. K. Singh and G. R. Thorpe, "Natural convection in a confined fluid overlying a porous layer — a comparison study of different models," *Indian Journal of Pure and Applied Mathematics*, vol. 26, pp. 81-95, 1995.
- [7] M. A. Sheremet and T. A. Trifonova, "Unsteady conjugate natural convection in a vertical cylinder partially filled with a porous medium," *Numerical Heat Transfer, Part A: Applications*, vol. 64, no. 12, pp. 994-1015, 2013.
- [8] M. A. Sheremet and T. A. Trifonova, "Unsteady conjugate natural convection in a vertical cylinder containing a horizontal porous layer: Darcy model and Brinkman-extended Darcy model," *Transport in Porous Media*, vol. 101, pp. 437-463, 2014.
- [9] F. Liu, B. Chen, and L. Wang, "Experimental and numerical estimation of slip coefficient in a partially porous cavity," *Experimental Thermal* and Fluid Science, vol. 44, pp. 431-438, 2013.

- [10] M. Mharzi, M. Daguenet, and S. Daoudi, "Thermosolutal natural convection in a vertically layered fluid-porous medium heated from the side," *Energy Conversion & Management*, vol. 41, pp. 1065-1090, 2000.
- [11] A. K. Singh, P. Agnihotri, N. P. Singh, and A. K. Singh, "Transient and non-Darcian effects on natural convection flow in a vertical channel partially filled with porous medium: Analysis with Forchheimer-Brinkman extended Darcy model," *International Journal of Heat and Mass Transfer*, vol. 54, pp. 1111-1120, 2011.
- [12] F. C. Lai and F. A. Kulacki, "Natural convection across a vertical layered porous cavity," *International Journal of Heat and Mass Transfer*, vol. 31, pp. 1247-1260, 1988.
- [13] R. Rana, R. N. Horne, and P. Cheng, "Natural convection in a multi-layered geothermal reservoir," *Journal of Heat Transfer*, vol. 101, pp. 411-416, 1979.
- [14] R. McKibbin and M.J. O'Sullivan, "Onset of convection in a layered porous medium heated from below," *Journal of Fluid Mechanics*, vol. 96, pp. 375-393, 1980.
- [15] D. Gobin, B. Goyeau, and A. Neculae, "Convective heat and solute transfer in partially porous cavities," *International Journal of Heat and Mass Transfer*, vol. 48, pp. 1898-1908, 2005.
- [16] A. C. Baytas, A. F. Baytas, D. B. Ingham, and I. Pop, "Double diffusive natural convection in an enclosure filled with a step type porous layer: Non-Darcy flow," *International Journal of Thermal Sciences*, vol. 48, pp. 665-673, 2009.
- [17] A. Bagchi and F. A. Kulacki, "Natural convection in fluid superposed porous layers heated locally from below," *International Journal of Heat and Mass Transfer*, vol. 54, pp. 3672-3682, 2011.
- [18] A. Bagchi and F. A. Kulacki, "Experimental study of natural convection in fluid-superposed porous layers heated locally from below," *International Journal of Heat and Mass Transfer*, vol. 55, pp. 1149-1153, 2012.
- [19] F. Arpino, A. Carotenuto, N. Massarotti, and A. Mauro, "New solutions for axial flow convection in porous and partly porous cylindrical domains," *International Journal of Heat and Mass Transfer*, vol. 57, pp. 155-170, 2013.
- [20] A. A. Merrikh and A. A. Mohamad, "Non-Darcy effects in buoyancy driven flows in an enclosure filled with vertically layered porous

media," International Journal of Heat and Mass Transfer, vol. 45, pp. 4305-4313, 2002.

- [21] F. H. Harlow and J. E. Welch, "Numerical calculation of time-dependent viscous incompressible flow," *Physics of Fluids*, vol. 8, pp. 2182-2189, 1965.
- [22] G. S. Beavers and D. D. Joseph, "Boundary conditions at a naturally permeable wall," *Journal of Fluid Mechanics*, vol. 30, pp. 192-203, 1967.
- [23] G. Neale and W. Nader, "Practical significance of Brinkman's extension of Darcy's law: Coupled parallel flows within a channel and a bounding porous medium," *Canadian Journal of Chemical Engineering*, vol. 52, pp. 475-478, 1974.
- [24] M. A. Sheremet, "Laminar natural convection in an inclined cylindrical enclosure having finite thickness walls," *International Journal of Heat* and Mass Transfer, vol. 55, pp. 3582-3600, 2012.
- [25] M. A. Sheremet, "Unsteady conjugate thermogravitational convection in a cylindrical region with local energy source," *Thermophysics and Aeromechanics*, vol. 18, no. 3, pp. 447-458, 2011.



Mikhail A. Sheremet was born in Russia. He received his PhD (Russia, Candidate of Science in Physics and Mathematics) in 2006, Tomsk State University and habilitation (Russia, Doctor of Science in Physics and Mathematics) in 2012, Tomsk State University, Russia.

He is a professor at the Department of Theoretical Mechanics, Faculty of Mechanics and Mathematics, Tomsk State University, Russia. Now he also works as the

head of the Laboratory of Convective Heat and Mass Transfer, Tomsk State University, Russia. His recent research concentrates on conjugate heat and mass transfer; natural and mixed convection; heat and mass transfer in porous media; fluid flow and heat transfer in nanofluids; turbulence models; radiation heat transfer; heat transfer and flow pattern in electronic systems.