Prediction of Nanofluid Forced and Mixed Convection Heat Transfer through an Annular Pipe

F. Benkhedda, T. Boufendi, and S. Touahri

***Abstract*—This work is a numerical simulation of the 3D forced and mixed convection heat transfer of Al_2O_3–water nanofluid flow through an annular pipe. The interest of this research is in enhancing heat transfer by using a nanofluid instead a usual fluid without solid particles. The external pipe is uniformly heated while the inner cylinder is insulated. Based on the single approach, the conservation equations are solved by a second order precision finite volume method. Extensive results are obtained for different values of the Reynolds (500–2000) and Grashof (0, 10^4, 10^5) numbers and the nanoparticle concentration (1, 4, 8%). Our results show that the mixed convection Nusselt number becomes more superior to that of the forced convection when the Grashof number is increased. Furthermore, when the Reynolds number is fixed, the temperatures undergo a circumstantial variation under the influence of the Grashof number with significant azimuthally variation. Also, for the same concentration of nanoparticles, temperatures within the nanofluid are strongly influenced by the Reynolds number. They decrease with increasing Reynolds number.**

***Index Terms*—Nanofluid, convection heat transfer, annular duct, numerical prediction.**

I. Introduction

Enhancement of the thermal characteristic of liquid has been achieved by adding micrometer particles to a base fluid, Maxwell [1]. These micron-sized particles cause some problems such as erosion, clogging, rapid sedimentation, and high-pressure drop, all these problems have been solved by using solid nano particles dispersed uniformly and suspended stably in conventional liquids. This fluid was termed a "nanofluid" by Choi [2] in 1995 to characterize the new class of fluids with superior thermal properties to prevalent base fluids. Nanoparticles used in nanofluids have been made of various materials, such as oxide ceramics (Al_2O_3, CuO), carbide ceramics (SiC, TiC), metals (Cu, Ag, Au), semiconductors (TiO_2, SiC), and carbon nanotubes. Also, many types of liquids, such as water, ethylene glycol (EG), and oil, have been used as base liquids in nanofluids. The volumetric fraction of the nanoparticles is usually below 5 % with respect which can provide effective improvements in the thermal conductivity and convective heat transfer of base fluids. Roy *et al*. [3] investigated numerical study of laminar flow heat transfer for (Al_2O_3–EG) and (Al_2O_3–water) and reported an improvement in heat transfer rate. Also they showed that wall shear stress increases with increasing nanoparticles concentration and Reynolds number. Despite the fact that nanofluid is a two phase mixture, since the solid particles are very small size they are easily fluidized and can be approximately considered to behave as a fluid Xuan *et al*. [4]. Therefore, considering the ultrafine ($< 100nm$) and low volume fraction of the solid particles, it might be reasonable to treat nanofluid as single phase flow in certain conditions, Yang *et al*. [5]. As this approach is simpler to use several theoretical studies were done based on this approach [6]. Mixed convection heat transfer in tubes appears in many industries such as heat exchangers and solar energy collectors and several works have been developed for the nanofluid behaviours, Behzadmehr *et al*. [7]. Also, the annuluses are a common and important geometry for thermofluid device enhancement. We can cite, among others, the works of Moghari *et al*. [8] who studied laminar mixed convection in horizontal annulus with constant heat flux at the inner and outer walls and Izadi *et al.* [9] who investigated 2D laminar forced convection of a nanofluid consisting of (Al_2O_3–water) numerically in a annulus with single phase approach.

In this study, we treat the single phase fluid model in the annulus geometry by highlighting the influence of parameters related to the convection modes and concentration of solid particles.

TABLE I: Thermophysical Properties

Physical quantity	Fluid phase water	Alumina Al_2O_3
μ (Ns/m^2)	8.91 10^{-4}	-
C_p (J/kgK)	4179	765
ρ (kg/m^3)	997.1	3970
k (W/mK)	0.613	40
β (K^{-1})	21 10^{-5}	0.85 10^{-5}
d_p (nm)	0.384	47

II. The Geometry and Mathematical Model

The problem of study is 3D steady, laminar forced and mixed convection of a nanofluid flow (Al_2O_3–water) in a long horizontal annular pipe of length L formed by two concentric cylinders, inner radius R_i and outer radius R_o. The outer cylinder is heated by an imposed uniform heat flux while the inner cylinder is adiabatic. Fig. 1 shows one half of the geometry of the considered problem. The nanofluid with single-phase approach is presented at the entrance by a constant velocity V_0 and a constant temperature, T_0. Dissipation and pressure work are neglected in order to be able using single-phase approach. It is assumed that the fluid

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phase and solid nanoparticles are in thermal equilibrium with zero relative velocity, [10]. So, single phase fluid procedure can be applied and the Boussinesq approximation is adopted. Table I presents thermophysical properties of base fluid and solid nanoparticles. Some simplifying suppositions are imposed, [11]: incompressible flow, no chemical reactions, dilute mixture (ϕ<<1) and negligible radiative heat transfer.

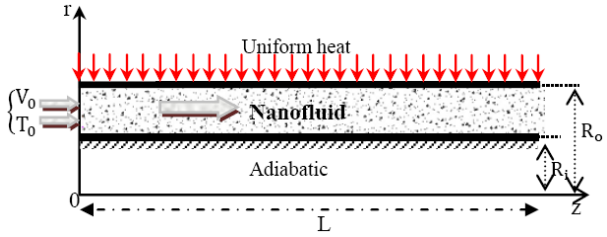

Fig. 1. The physical model and the correspondent geometry.

The physical principles involved in this problem are well modeled by the following non-dimensional conservation partial differential equation of mass, momentum and energy with their appropriate boundary conditions:

$$\frac{\partial \rho}{\partial t^*} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{V}^*) = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{V}^*}{\partial t^*} + \boldsymbol{\nabla} \cdot (\boldsymbol{V}^* \boldsymbol{V}^*) = -\boldsymbol{\nabla} P^* + \left[(1/Re_0)\left(\frac{\rho_f}{\rho_{nf}} \frac{1}{(1-\emptyset)^{2.5}}\right)\right][\boldsymbol{\nabla} \cdot \mu^* \boldsymbol{\nabla} \boldsymbol{V}^* + Gr0Re0\,{}^2 1-\emptyset(\rho\beta)f+\emptyset(\rho\beta)s\beta f\times 1-\emptyset\rho f+\emptyset\rho sT^* \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + \boldsymbol{\nabla} \cdot (\boldsymbol{V}^* T^*) = \left[(1/Re_0 Pr_0)\frac{(\rho C_p)_f}{(\rho C_p)_{nf}}\right] \cdot \left[\boldsymbol{\nabla} \cdot \left(\frac{\mathrm{k_{nf}}}{\mathrm{k_f}} \boldsymbol{\nabla} \mathrm{T}^*\right)\right] \tag{3}$$

The physical and thermal nanofluid properties are calculated using different appropriate formulae available in the literature. The density ρ_{nf}, the heat capacity $(\rho C_p)_{nf}$ and the thermal expansion coefficient $(\rho\beta)_{nf}$:

$$\rho_{nf} = (1-\emptyset)\rho_f + \emptyset\rho_s \tag{4}$$

$$(\rho C_p)_{nf} = (1-\emptyset)(\rho C_p)_f + \emptyset(\rho C_p)_s \tag{5}$$

$$(\rho\beta)_{nf} = (1-\emptyset)(\rho\beta)_f + \emptyset(\rho\beta)_s \tag{6}$$

The nanofluid thermal conductivity is calculated from the Hamilton Crosser equation [12]:

$$k_{nf} = \left(\frac{k_s+(n-1)k_f-(n-1)\varphi(k_f-k_s)}{k_s+(n-1)k_f+\varphi(k_f-k_s)}\right)k_f \tag{7}$$

(for the spherical particles: n=3)

The Brinkman model [13] is used for the evaluation of the nanofluid viscosity:

$$\mu_{nf} = \frac{\mu_f}{(1-\emptyset)^{2.5}} \tag{8}$$

In all these relationships, $\emptyset$ are the volume fraction and the subscripts f and s are associated with fluid and solid nanoparticles successively.

This set of nonlinear governing equations has been solved subject to following boundary conditions:

At the inlet of the duct: $(z^* = 0)$

$$R_i{}^* \le r^* \le R_e{}^* \text{ and } 0 \le \theta \le 2\pi :$$

$$u^* = w^* = 0,\ v^* = T^* = 1 \tag{9}$$

At the outlet of the duct : $(z^* = L^*)$

$$R_i{}^* \le r^* \le R_e{}^* ; 0 \le \theta \le 2\pi :$$
$$\frac{\partial u^*}{\partial z^*} = \frac{\partial w^*}{\partial z^*} = \frac{\partial v^*}{\partial z^*} = \frac{\partial}{\partial z^*}\left(\frac{\partial T^*}{\partial z^*}\right) = 0 \tag{10}$$

(The duct length L is 200 time of the hydraulic diameter D_h to insure that the fully developed condition is reached at the outlet)

At the outer wall of the inner cylinder: $r^* = R_i{}^*$
$0 \le \theta \le 2\pi$ and $0 \le z^* \le L^*$ $\quad u^* = w^* = v^* = 0$ and

$$\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=\mathrm{r_i}^*} = 0 \tag{11}$$

At the outer wall of the outer cylinder: $r^* = R_\mathrm{o}{}^*$
$0 \le \theta \le 2\pi$ and $0 \le z^* \le L^*$ $\quad u^* = w^* = v^* = 0$ and

$$\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=r_\mathrm{o}{}^*} = \frac{k_f}{k_{nf}} \tag{12}$$

Along the angular direction, the periodic conditions are imposed.

The heat transfer is notified by the Nusselt number, which reflects the relative ration of convective to conductive heat transfer. Since the surface of the inner cylinder is adiabatic, the Nusselt number will be reported to the outer surface of the outer cylinder.

At steady state, the local Nusselt number depending on angular and axial position is expressed by the following equation:

$$Nu(\theta, z^*) = \frac{h_o(\theta,z)D}{k_f} = \left[\frac{(k_{nf}/k_f)(\partial T^*/\partial r^*)|_{r^*=1}}{T^*(1,\theta,z^*)-T^*{}_b(z^*)}\right] \tag{13}$$

where the dimensionless bulk fluid temperature is:

$$T_b^*(z^*) = \frac{\int_{R_i^*}^{R_e^*}\int_0^{2\pi} V^*(r^*,\theta,z^*)T^*(r^*,\theta,z^*)r^* dr^* d\theta}{\int_0^{\frac{1}{2}}\int_0^{2\pi} V^*(r^*,\theta,z^*)r^* dr^* d\theta} \tag{14}$$

The local axial mean peripheral Nu number is:

$$Nu(z^*) = \frac{1}{2\pi}\int_0^{2\pi} Nu(\theta, z^*)d\theta \tag{15}$$

and the average Nu for the whole interface is :

$$Nu = \frac{1}{100}\int_0^{100} Nu(z^*)dz^* \tag{16}$$

III. Numerical Resolution

This set of coupled non-linear differential equations was discretized by the finite volume method, Patankar [14]. The temporal discretization of the derivation terms follows the backward Euler scheme whereas the convective and the non-linear terms follow the Adams-Bashfort scheme whose the truncation error is of Δt^{*2}. The spatial disretization of the diffusive terms and the pressure gradient follows the fully implicit central difference scheme. The systems of the linearized algebraic equation obtained are solved by the SIMPLER algorithm. With step time of $\Delta t^* = 10^{-3}$, the time marching is continued until the steady state is reached. The convergence is confirmed by the satisfaction of the global mass and energy balances. In the r^*, θ, z^* directions, tests on

the influence of the mesh allowed to retain the following mesh 26 × 44 × 162 successively. A validation concerning the forced convection is verified by the comparison of our results with those of Nazrul and al. [15]. The results concern the axial Nusselt number at the interface of the external cylinder and the fluid for the forced convection case. The comparison shows a good agreement in Fig. 2. The numerical code used, is a transformation of the code developed in the first step by Boufendi and Afrid [16] and in the second step by Touahri and Boufendi [17]–[19].

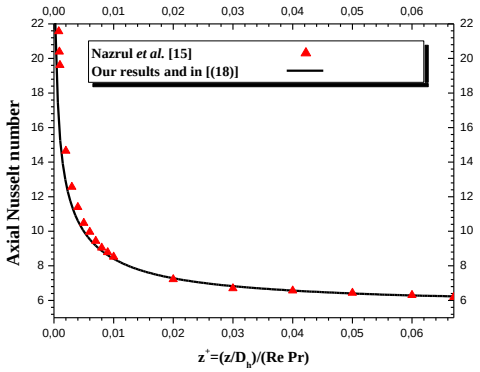

Fig. 2. Axial evolution of the circumferentially mean axial Nusselt number; A comparison with the results of [15].

IV. Results and Discussion

The results are obtained for different Reynolds numbers (500 to 2000), different concentration (1,4 and 8%) and different Grashof numbers (0, 10^4, 10^5). For the brevity of the paper, we have chosen to limit for a 4% concentration.

A. Hydrodynamic and Thermal Fields

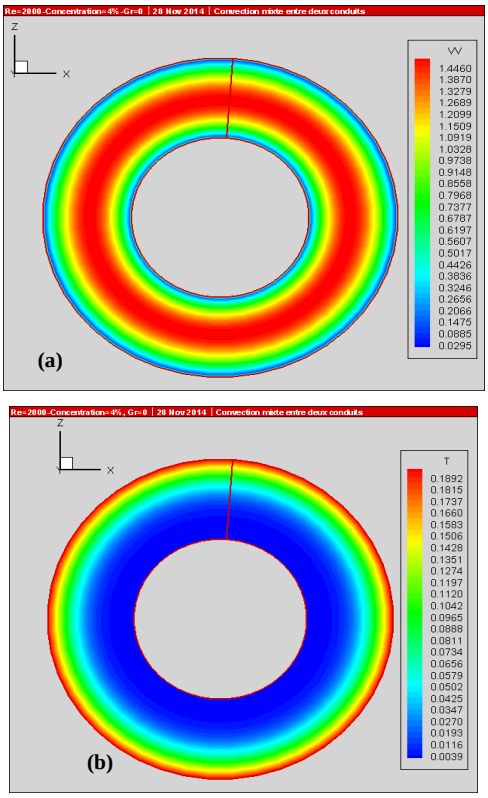

Fig. 3. The isolines velocities (a) and the isotherms (b) for the nanofluid at the exit in forced convection mode ($Re = 2000, Gr_{nf} = 0, Pr = 6.69;\ \tau = 4\%$).

The hydrodynamic (a) and the thermal fields (b) are illustrated in Fig. 3 and Fig. 4 for the forced (Gr=0) and mixed cases (Gr=10^4, 10^5) at the exit duct. In the forced regime (a), the velocity distribution show a central area where it is high and areas where velocities are low located on either side of this central part.This velocity distribution obeys a parabolic velocity profile which is characteristic of a hydrodynamically developed state. In all the cases studied, this profile is quickly reached near the entrance where the axial velocity assumes a maximum value at the center of the annulus which is about 1.476.

From topographically viewpoint, the iso velocities are concentric circles. Thermal fields are also shown in Fig. 3 and 4 for the two cases (b). The topography of the thermal fields shows for the case Fig. 3(b) that the isothermal surfaces are concentric circles whose temperature variation decreases from the outer wall to adiabatic. In all cases the maximum temperatures are on the outer pipe and minimums are on the inner conduit. In the Fig. 4(b), these profiles clearly illustrate the influence of natural convection by the deformation of the isotherms which are almost flattened in the entire upper part of the annular space. These different qualitative and quantitative variations are characteristic of the mixed convection in a pipe since the gradients of angular temperatures are not zero. However, two other important points emerge through our results: (i) For the same concentration of nanoparticles, temperatures within the nanofluid are strongly influenced by the Reynolds number. They decrease with increasing Reynolds number. (ii) For the same Reynolds number, temperatures undergo circumstantial change with concentration. They increase substantially with increasing concentration.

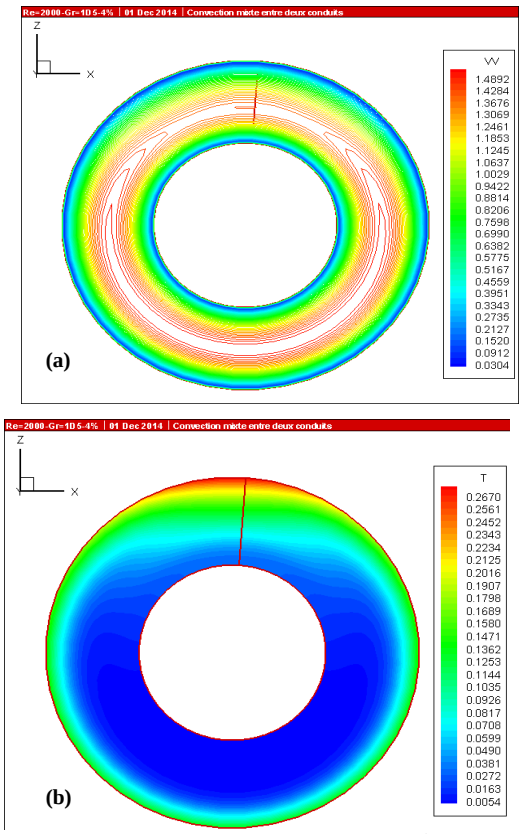

Fig. 4. The isolines velocities (a) and the isotherms (b) for the nanofluid at the exit in mixed convection ($Re_{nf} = 2000, Gr_{nf} = 10^5, Pr_{nf} = 6.69;\ \tau = 4\%$).

B. Heat Transfer

The heat transfer is illustrated with the Nusselt numbers for the forced and mixed convection cases. For the convection mode, Fig. 5 shows the variation of the Nu along the duct for different Reynolds numbers. It is clear that these variations with abrupt decrease in the short entrance zone and a very slow diminution and asymptotic, constant at the large exit zone is physically acceptable with the same behaviour for a fluid flow in forced convection. In contrast, the Fig. 6 illustrates perfectly the effect of the increase of the Grashof number on the evolution of the nanofluid along the pipe. For a same Reynolds number and a same concentration of nanoparticles the Nusselt number increase with the increasing of the Grashof number.

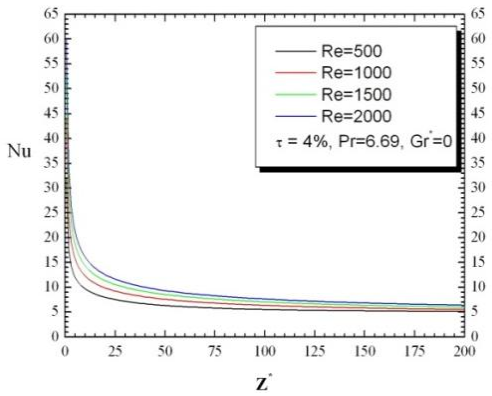

Fig. 5. The Nusselt number profiles along the annulus for the forced convection case.

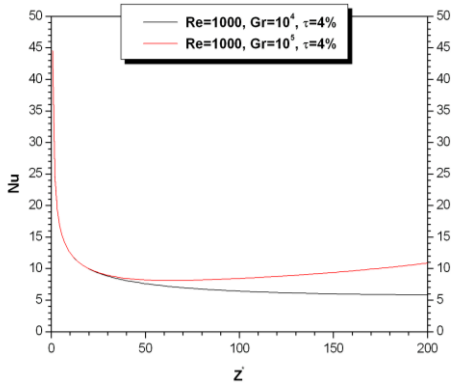

Fig. 6. The Nusselt number profiles along the annulus for the mixed convection case.

V. Conclusion

This work is a numerical simulation of convective heat transfer in nanofluid flowing through an annulus formed by two horizontal concentric cylinders. The inner cylinder is adiabatic while the outer cylinder is subjected to constant parietal heating. The results can be synthesized as follow: when the concentration is fixed, the temperature within the nanofluid is strongly influenced by the Reynolds number. They decrease with increasing Reynolds number. Whereas for the same Reynolds number, temperatures undergo circumstantial change with concentration. Also, by the influence of the Grashof number, it is seen that very near the inlet, the variation of the temperature of the interface is similar to that of the forced convection. Under the effect of natural convection, the azimuthally variation of the temperature at the interface becomes large. The increase Grashof increases the heat transfer quantified by growth Nusselt number. This work confirmed the increased heat transfer when using a nanofluid instead of a single fluid. In addition to the natural effect of the mixed convection which promotes mixing of fluid layers, there is also an improvement in heat transfer due to the nature of nanofluid by the addition of the thermal conductivity of the solid particles which increases the overall thermal conductivity of nanofluid.

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