# Tolerance Design of Robot Parameters Using Generalized Reduced Gradient Algorithm

Trang Thanh Trung, Li Wei Guang, and Pham Thanh Long

Abstract-In robot design, how to allocate tolerances for parts in manufacturing and assembling of robot is very important because this directly affects product quality and manufacturing cost. This paper introduces a technique using the Generalized Reduced Gradient algorithm optimization to allocate tolerances into robot parts. This method consists of three steps. First, based on the particular structure of robot, various methods are considered before the best method suitable for modeling the associated equation is chosen. Then, a mathematical model for tolerance allocation is formulated and transferred into the non-linear multi-variable optimization problem. Finally, this optimization problem is solved by using the Generalized Reduced Gradient algorithm. Two examples are used to verify the feasibility of the proposed method; the accuracy and effectiveness of the proposed method in producing the tolerance allocations is also illustrated via calculation and simulation results.

*Index Terms*—Industrial robot, tolerances, optimization problem, generalized reduced gradient algorithm.

## I. INTRODUCTION

Robots are usually employed in industrial and medical applications to position or orient an object, where high accuracy, repeatability and stability of operations are required. The repeatability is a measure of the ability of the robot to move back to exactly the same pose over and over again, while accuracy is defined as the ability of the robot to precisely move to a desired pose in 3-D space [1]. Unacceptable performance, which is the positional and directional deviation of the robot end effector, may be caused by a number of sources such as the joint clearance of actuators and controllers, manufacturing and assembling errors, different types of measurement and control errors, elastic deformations of structural components, and so on [2]-[4]. As a matter of the fact, all these errors are random in nature and especially there is no way of eliminating the dimensional tolerances and clearances prescribed on the manufacturing and assembling operations [5]. Hence the requirement of selecting tolerance within the smallest possible is emerged.

The smaller the tolerance, the better the product quality but the higher the manufacturing cost. It is always not an easy task to choose from a proper balance between the product quality and the manufacturing cost [6]. Traditionally, these parameter tolerances are mostly selected by experience and intuition of designers, handbooks, and standards, which lead to some errors [7]. So the product quality is not guaranteed and the manufacturing cost may be higher than necessary. On the other hand, designing robot to satisfy the desired performance requirement is a complex activity because of the nonlinear and coupled relationship between the robot actuators and end-effectors and uncertainties presenting in kinematic parameters. These uncertainties attribute to error factors in robot systems, which cause variability in performance. Therefore, how deviation of link dimensions and joint tolerances in the robot systems contributing to the end-effector deviation and what analysis method being used to evaluate the performance are the crucial issues.

In this paper, a technique using Generalized Reduced Gradient (GRG) optimization algorithm to solve the tolerance design robot problem is proposed. This proposed method can be applied to both robot arm and parallel robot, and its procedure is described with two processes: direct and inverse. The results of two examples simulation are presented to demonstrate the accuracy and effectiveness of this method.

## II. LITERATURE REVIEW

As the aforementioned, in robot design selection of optimum design and process parameter tolerance of robot is a challenging task. In order to solve this problem, a number of researches have been developed using techniques as conventional optimization methods, quality engineering methods, genetic algorithms (GA), simulated annealing (SA), neuro-fuzzy learning, and so on and some are already in regular use, statistical planning of experiments, numerical simulation procedures, and probabilistic modeling.

Many researchers consider tolerance allocation as an optimization problem in which the tolerance values of parts are taken as the control variables, and the machining costs are taken as the objective function to be minimized. Michael and Siddall extended the conventional design optimization problem, in which the nominal values of the design variables are of interest, to include the optimal allocation of manufacturing tolerances [8]. Parkinson provided an application in which tolerances of a system is selected using optimization technique [9]. In Weidong Wu, S. S. Rao's research, they focus on the optimal allocation of joint tolerances with consideration of the positional and directional errors of the robot end effector and the manufacturing cost

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[3]. Similar Rao R. S *et al.* [10], the interval analysis is used for predicting errors in the performance of robot. By using Denavit-Hartenberg (DH) rule for modeling the kinematic problem, the objective functions include the dimensional tolerances (d in DH table) and assembly errors (in DH table). These errors, however, have no relationship with the level of free movement along the centripetal direction of the joints which can lead to the choice of accurate level of bearing. The unknown joint variables are modeled as interval parameters due to the inherent uncertainty. The cost-tolerance model is assumed to be of an exponential form during optimization, in optimum status this value reach the minimum. However, in their work they have not yet mentioned the way of identifying the tolerances of each joint, as well as the application on parallel robot.

The optimization-based tolerance approach methods tend to be impractical as the complexity of the robot assemblies increases. Gadallah and ElMaraghy [11] are one of the first researchers attempting to apply the parameter design techniques of quality engineering to the tolerance optimization problem through the concept of the quality loss function. Many researchers, like Choi H-GR, Park M-H [12], Cho B-R, Kim YJ [13] and Feng C-X, Wang J [14] found in their studies the concept of loss function in the tolerance allocation problem. An extensive review of the methodologies to obtain robust design of products that have low performance variation caused by the variations of control factors and noise factors is given by Rout and Mittal [15].

Another group putting efforts on overcome the impracticality of optimization-based tolerance allocation problems give birth to some novel approaches based on relatively new techniques, such as GA, neural networks, SA and fuzzy logic [16]. Paredis and Khosla proposed a distributed agent-based GA approach to create fault tolerant serial chain manipulators from a small inventory of links and modules [17]. Coello et al. developed new technique that combines GA and the weighted min-max multi-objective optimization method for robot design [18]. Zhuang et al. applied GA to select the optimal robot measurement configurations, which is an important element in robot calibration [19] while Ji S. et al. [20], [21] and Chen T-C and Fischer G. W. [22] applied the GA to the tolerance allocation problem. Later Zhang. D et al. implemented GA to obtain the optimum design of parallel kinematic tool-heads considering the global stiffness and workspace volume [23]. Subsequently a method based on GA was introduced by P.T. Zacharia and N. A. Aspragathos for the determination of the optimal sequence of a non-redundant manipulator end-effector, considering multiple configurations [24]. On the other hand, Kopardekar and Anand [25] applied neural network techniques to tolerance allocation. While Dupinet E. et al. [26] exploited fuzzy logic and SA for the purpose of efficiently dealing with the machine capability and manufacturing specific phenomena, such as mean shift, the back propagation method is used to train the network that generates the part tolerances. The SA was discussed for optimum kinematic design of serial link manipulators [27] and it was adopted to obtain near optimal measurement configurations of robot for calibration [28].

Due to the random nature of link dimensions and joint

clearances, Jeong Kim et al. proposed a stochastic approach to figure out the reliability for the open-loop mechanism [5]. In their work, with the assumption that all kinematical parameters are the normally distributed random variables, the stochastic model of the links with dimensional tolerances and of the revolute joints with clearances is presented. The kinematical reliability for the positioning and orientation repeatability is then calculated analytically based on an advanced first-order second moment method. Bhatti and Rao developed a probabilistic approach combined with Monte Carlo simulation method to the manipulator kinematics and dynamics taking into account the relationships among the geometric tolerances, arm configuration and manipulator reliability [29], [30]. Lee and Woo [31] formulated the tolerance synthesis as a probabilistic optimization problem in which a random variable is associated with a dimension and its tolerance. A general probability density function of the endpoints of planar robots based on probability theory was established by Zhu and Ting [32] to find the probability of the robot end point locations with a desired tolerance zone and to determine the joint clearance value. Furthermore, they offered a kinematical model to understand the effect of joint clearances and to determine the directional deviation of single and multiple degree-of-freedom linkages against the worst case [33]. By using probabilistic approach, Rao and Bhatti proposed manipulator reliability to express its kinematic and dynamic performance. Where the manipulator reliability is defined as probability of end effector pose falling within a specified range from the desired pose [34].

An alternative method regularly used is the statistical experiments. Riemer and Edan observed experimentally that there is a statistically significant difference among the repeat abilities at both different locations in the workspace and the different height of the target point [35]. Parametric tolerance design of a manipulator which used full factorial design of experiment (DOE) approach without taking noise factor into consideration had been attempted by B.K. Rout and R.K. Mittal [36]. They had applied the Taguchi method to find the optimal parameters settings for improving the quality of performance of a manipulator [37]. They also has been proposed a hybrid technique which combines the evolutionary optimization technique and orthogonal array of the Taguchi method to optimize the design parameter tolerances such as link dimensions, link inertias, and actuating torque fluctuations. These parameter tolerances actually would deliver specified level of performance measure with minimum manufacturing cost. The hybrid approach proposed is the best choice for the purpose of parameter tolerance design considering the effect of noises for performance simulation [4], [38]. However, this technique is an off-line procedure and among the manufacture errors, it can solve only the tolerances of link dimensions which are the geometrical error while the problem of determining the limited tolerances of the dynamic joints in robot is not considered.

For the optimal design of robot parameter tolerance, some researchers also used the approach based on Jacobian matrix. Maciejewski investigated the pose in which a redundant serial manipulator would have optimum dexterity in case of jam in any of its joints by using the singular values of the Jacobian matrix [39], [40]. Lewis and Maciejewski defined a fault tolerance measure for joint jam in the redundant serial manipulators based on the smallest singular value of the Jacobian matrix [41]. Later, Mahir Hassan and Leila Notash applied a method based on the Jacobian matrix to determine the joint tolerance which satisfies both economic and technology objectives. In this work, the type and the potential locations of the redundant backup joints are assumed to be pre-identified and their axes directions are identified by employing the Lagrange multiplier optimization method [42]. This solution therefore can be applied only to the fully actuated robot but the parallel robot where the tolerances of several passive joints cannot be solved by this solution. Moreover, it is complicated to calculate the optimal manufacture tolerance based on Jacobian matrix because of using the Lagrange multiplier method for eliminating the cross-affect among the tolerances. Recently, Phan Bui Khoi et al. has been used an approach based on the differential of Jacobian matrix and GA method to identify the positional and directional errors of the end effector along with the geometrical errors of the intermediate links and joints. The inverse mechanism of the tolerance problem however has not been mentioned in this approach [43].

For the investigation of the relationship between the pre-identified errors in the component links and the positional and directional accuracy in the end effector, Miomir Vukobratovic and Branislav Borovac introduced a map error index method (MEI). By changing links parameters deviations, a whole family of portraits with the accompanying deviations may be obtained [44]. For the purpose of evaluation of the robot kinematic accuracy through the computation of its responses with and without clearance, a

coefficient of performance 
$$MEI = \sqrt{\frac{\sum_{i=1}^{k} \left(\frac{1}{\text{index\_value}}\right)^2}{k}}$$
 is drawn

as a figure illustrating the robot response in the form of error chart. Based on this information obtained, in the stage of the robot assembly, the selection of the links with appropriate deviations will be performed to ensure a desired accuracy of the robot end effector. It is relatively easy for investigation procedure but the inverse mechanism, the effect of the end-effector deviation on both tolerances of link dimensions and deviations of joints has not been showed.

From the discussions above, it can be concluded that there are many available efficient methods applying to robot issues but the problem of identifying the tolerances of links dimension and joints applying in robot arm and parallel robot with both direct and inverse directions is occasionally considered. Hence, a new approach relating to the determination of tolerances of links dimension and joints satisfying the mentioned objectives is proposed in this paper. In which, the tolerance allocation including the tolerances of the links and joints are represented as an optimization problem and a new mathematical model based on RosenBock Banana function is established and solved by using the Generalized Reduced Gradient algorithm. The tolerance allocation for the robot arm and parallel robot industrial assembly is produced by the above method, and the results show that the method can be used in design with the optimal tolerance values of parts.

## **III.** THE FORMATION OF THE OPTIMAL PROBLEM

The associated equations are the fundamental basis to form the kinematic problems of robots. In building associated equations, general principles are based on closed-loop vectors. Depending on each particular structure of robots, the inter-relations of reference systems can be modeled in the form of closed-loop vectors. The diagram of closed-loop vectors in both robot arm and parallel robot is illustrated in Fig. 1.



Fig. 1. Diagram of closed-loop vectors in robot arm and parallel robot.

The relationship is written as the loop vector equation as follows:

For parallel robot:

$$A_1 A_2 \dots A_n = I \tag{1}$$

For robot arm:

$$A_1 A_2 \dots A_n = X . E . R . T^{-1}$$
 (2)

The matrices on the left hand side of these equations are determined by the DH rule, screw transpose rule or geometric rule depending on the selection of the users. The right hand side matrices illustrate the position and direction of the end effector and they are determined based on the control trajectory of robot in space. The expansion the equation (1, 2)can be rewritten as:

$$\begin{vmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{vmatrix}$$
(3)

where: *n*, *s*, *a* are direction vectors; *P* is location vector;  $a_{ii}$ with *i*,  $j = 1 \div 3$  is the direction cosine;  $a_{14}$ ,  $a_{24}$ ,  $a_{34}$  are elements projected onto the global coordinate system Oxyz as P's components, respectively.

Due to the orthogonal feature of the orient vectors, elements in (3) can be transformed to equation (4). This equation is the inverse kinematic problem of industrial robot.

$$\begin{cases} s_x = a_{12} \\ a_x = a_{13} \\ a_y = a_{23} \\ p_x = a_{14} \\ p_y = a_{24} \\ p_z = a_{34} \end{cases} \begin{pmatrix} s_x - a_{12} = 0 \\ a_x - a_{13} = 0 \\ a_y - a_{23} = 0 \\ p_x - a_{14} = 0 \\ p_y - a_{24} = 0 \\ p_z - a_{34} = 0 \end{cases}$$
(4)

Squaring two sides of (4) and add each side has:

$$(s_{x} - a_{12})^{2} + (a_{x} - a_{13})^{2} + (a_{y} - a_{23})^{2} + (p_{x} - a_{14})^{2} + (p_{y} - a_{24})^{2} + (p_{z} - a_{34})^{2} = 0$$
(5)

It is clear that (5) is always  $\ge 0$  so the minimum of (5) is 0. Denote *T* as the objective function in the left hand side:

$$T = (s_x - a_{12})^2 + (a_x - a_{13})^2 + (a_y - a_{23})^2 + (p_x - a_{14})^2 + (p_y - a_{24})^2 + (p_z - a_{34})^2$$
(6)

This function form is well known as Rosenbrock-Banana function [45].

The goal of kinematic control is to ensure the accuracy of the position and direction of the robot end effector. Therefore, it is necessary to determine the value of the joints to meet the requirements of minimum deviation in positions and directions of the end effector as well as to meet the constraint conditions in robot structure.

Denote  $f = (q_1, q_2, ..., q_n)$  is the vector of joint variables. Space D determines the value range of the joint variables:

$$\begin{cases} L_{1} \leq q_{1} \leq U_{1} \\ L_{2} \leq q_{2} \leq U_{2} \\ \vdots \\ L_{n} \leq q_{n} \leq U_{n} \end{cases}$$

$$(7)$$

where:  $L_i$  and  $U_i$  are constraints of joint variables.

T = f(q): function describing the deviation of position and direction of the robot end-effector.

The problem of determining the value of the joint variables is written as follows:

$$\begin{cases} T = f(q_1, q_2, ..., q_n) \rightarrow \text{minimize} \\ L_i \le q_i \le U_i \\ q_i \in D; i = 1 \div n \end{cases}$$
(8)

This is a mathematical model of the non-linear multi-variable optimization problem.

#### IV. SOLUTION METHOD FOR THE OPTIMIZATION PROBLEM

The non-linear optimization problem with general constraints is defined as follows:

$$\begin{cases} \operatorname{Min} f(x), x \in F \subset S \subset R^{n} \\ h_{i}(x) = 0, i = 1, 2..., p \\ g_{j}(x) \leq 0, j = p + 1, ..., q \\ l_{k} \leq x_{k} \leq u_{k}, k = 1, 2..., n \end{cases}$$
(9)

While this optimization problem could be solved by several methods as Sequential Quadric Programming method (SQP), GA, GRG, and so on, the function f(x) in this the optimization problem is the Rosenbrock-Banana function which has complex geometric representation. It is consequently best solved by GRG method [46], [47].

The GRG method is one of the techniques that are based on extending optimization methods for linear constraints applying to nonlinear constraints. This procedure is based on the idea of elimination of variables using the equality constraints. The goal of GRG is converting the constrained problem into an unconstrained one by using direct substitution. The development of the GRG method is followed by that of constrained variation. The approach used in GRG method is both determining an improved direction for the technical model and satisfying the constraint equations [48]. An example of Generalized Reduced Gradient (GRG2) algorithm for optimizing nonlinear problems could be found in Microsoft Excel Solver. This algorithm was developed by Leon Lasdon in University of Texas, and Allan Waren, of Cleveland State University (Microsoft Inc. 2011).

## V. DETERMINATION OF THE TOLERANCE OF JOINT ANGLE MOVEMENT

For quantitative assessment of joint manufacturing tolerance, this variable must be described in the form of mathematical model. The joint tolerance is resulted from the allocation of position and direction errors of the end-effector to the build-up joints in the procedure of design. Hence, the model of dynamic joint tolerance must describe the relationship among position and direction tolerances of the end-effector the geometric characteristics of the mechanism and the tolerances of build-up joints.

From the robot kinematic equation formulated as:

$$f_i(q_1, q_2, ..., q_6) = 0$$

$$i = 1 \div n$$
(10)

where n is the generalized coordinate sufficient to determine the position and direction of end-effector.

Define the current given position in the working space of robot as:

$$p_i = (x_i, y_i, z_i, \alpha_i, \beta_i, \gamma_i)$$
(11)

where:

 $(x_i, y_i, z_i)$  Describes the position of the end-effector;

 $(\alpha_i, \beta_i, \gamma_i)$  Describes the direction of the end-effector;

The value of joint variable in this condition could be found by solving the following set of equations:

$$f_i(q_1, q_2, \dots, q_6) = p_i$$

$$i = 1 \div n$$
(12)

Assuming that  $p_i$  in joint space at the given time is demonstrated by:

$$p_i = (q_1, q_2, \dots, q_6)^{(i)}$$
(13)



Fig. 2. The movement with the smallest step of moving platform between two points in space.

Considering a hexapod parallel robot in space with moving platform as shown in Fig. 2. Denoting dR as the smallest movement which is the design requirement of accurate performance. These moving quantities can be generalized as a sphere with the radial of dR and the center at O – the given point. The design requirement of this problem is that the joint tolerances will be determined from this limited deviation for the identification of free radial movement  $\delta$  of the built-up joints.

The tolerances of the built-up joints include radial clearance  $\delta$  and axial clearance  $\delta_1 + \delta_2$  (Fig. 3).



Fig. 3. Several types of clearance to be controlled in joints.

Choosing the smallest resolution axis among three axes x, y, z of the robot Oxyz coordinate for determining the smallest movement of moving platform from an arbitrary direction to dR sphere, then identifying the coordinates of the next point where the platform will move to as follows:

$$p_{i+1} = (x_{i+1}, y_{i+1}, z_{i+1}, \alpha_{i+1}, \beta_{i+1}, \gamma_{i+1})$$
(14)  
$$\left| p_{ix} + dx \right|$$

or: 
$$p_{i+1} = \begin{vmatrix} p_{iy} \\ p_{iy} + dy \\ p_{iz} + dz \end{vmatrix}$$

This coordinate will get the correspondent value of joint

variables when updating into equation (12) as:

$$p_{i+1} = (q_1, q_2, ..., q_6)^{(i+1)}$$
(15)

It means that every joint has performed a movement as:

$$\begin{cases} \delta q_1 = q_1^{(i+1)} - q_1^{(i)} \\ \vdots \\ \delta q_6 = q_6^{(i+1)} - q_6^{(i)} \end{cases}$$
(16)

They are the maximum movement of free angle in the range of manufacture deviation limitation. In this case, the calculation results will be converted to the accuracy of transmission system under the mechanism point of view when considering the design of Fig. 4.



Fig. 4. The transmission deviation of restricted angle caused by mechanic clearance defined by (16).

## VI. DETERMINATION OF THE DEVIATION OF LINK DIMENSIONS AND JOINT FREE RADIAL MOVEMENT BY USING INVERSE KINEMATIC

In Section V, a technique determining the free angle of the joints by using the inverse kinematic problem rather than Jacobian matrix method has been introduced. This method however has a drawback that the radial movements of tolerances in built-up links are unrevealed, so movement of the end-effector in a tolerance limit is unreliable. Besides, the small translation movements appear in Jacobian matrix when it has translation joints in robot.

The full kinematic model of the robot includes the length of built-up links in model (12) as follows:

$$f_i(q_1, q_2, ..., q_6, (l_i + \delta l_i)) = 0$$
  
(17)  
$$i = 1 \div n$$

where  $l_i$  is the nominal length of  $i^{th}$  link;  $\delta l_i$  is an unknown value called the allowable length deviation of the  $i^{th}$  link. The procedure of determining this value is:

Denote the given point in working space of robot as:

$$p_i = (x_i, y_i, z_i, \alpha_i, \beta_i, \gamma_i)$$
(18)

where:

 $(x_i, y_i, z_i)$  Describes the position of the end-effector;

 $(\alpha_i, \beta_i, \gamma_i)$  Describes the direction of the end-effector

The value of joint variable in this state can be found from the following equation:

$$f_i(q_1, q_2, ..., q_6) = p_i$$
  
 $i = 1 \div n$ 
(19)

Assuming that the point  $p_i$  in joint space at given time can be described by:

$$p_i = (q_1, q_2, ..., q_6)^{(i)}$$
 (20)

Combining (20) with the given nominal dimension value of each links  $l_i$ , equation (17) including *n* variables with the predetermined values  $\delta l_i$  when establishing the equation of small movements along the free movement of end-effector can be solved by GRG algorithm [46], [47].

When allocate the tolerance of end-effector to the built-up links inversely, the different ranges of calculated tolerance values of each joint are founded at different control points. The choice of tolerance range as in Fig. 5.

#### Nominal size x<sub>0</sub>



Domain selection tolerance Fig. 5. Tolerance choice of built-up links.



dL =  $\delta$ (b). Tolerance of joint clearance  $\delta 1$   $(\delta 1 + \delta 2) = dL$  $(L + \delta 2)$ 



Considering an arbitrary  $i^{th}$  link at the points from  $P_1$  to  $P_n$  in control trajectory, the tolerance range of  $i^{th}$  link changing around the value of nominal dimension can be calculated based on the small movement of the end-effector in inverse

kinematic problem. Defining set of the calculated values of link dimensions which are smaller than nominal dimensions is  $(x_1, x_2, ..., x_n)$ . Defining set of the calculated values of link dimensions which are larger than nominal dimensions is  $(x_1, x_2, ..., x_n)$ . For making the tolerance of the end-effector in the range of allowable values, the tolerance of  $i^{th}$  link should be chosen as: the lower limitation equals to max of  $(x_1, x_2, ..., x_n)$ , and the upper tolerance equals to min of  $(x_1, x_2, ..., x_n)$ .

From the calculated tolerance values of built-up links and the design requirements, the manufacture tolerances of both built-up links and joints will be determined as in Fig. 6.

### VII. THE EXAMPLE OF NUMERICAL SIMULATION

#### A. Robot Arm

Considering the kinematic diagram of robot Fanuc S900W in the form of equivalent open-series (Fig. 7), with DH table described in Table I.



Fig. 7. The equivalent kinematic diagram of robot Fanuc S900W.

TABLE	TABLE I: KINEMATIC PARAMETERS OF ROBOT FANUC S900W						
Joints	$R(z, \alpha)$	T(z, d)	T(x, a)	$R(x, \beta)$			
1	$(\alpha_1)$	650	0	90			
2	$(\alpha_2)$	0	700	0			
3	( <i>α</i> <sub>3</sub> )	0	0	90			
4	$(\alpha_4)$	675	0	-90			
5	$(\alpha_5)$	0	0	90			
6	$(\alpha_6)$	300	0	0			



Fig. 8. Six allowable moving points of the end-effector in the limited deviation range of a sphere.

Requirement: Determine the manufacture tolerances with the dimensions of 650, 700, 675, 300 in DH table, the given condition of uncontrolled deviation of arm center, the end-effector, is no higher than 0.2mm in all directions in working space (dR = 0.2mm, Fig. 8).

In Fig. 8 are the six allowable moving points of the end-effector in the limited deviation range of a sphere. The coordinates of these points can be calculated from the sphere center (the desired nominal position of robot end-effector center) and the given diameter of the sphere (the limited tolerance of the end-effector).

Considering the study points in the working space of robot, the inverse kinematic problem and applying the equations from (14) to (19), the study tolerances of the built-up links of robot Fanuc S900W are presented in Table II.

Applying the choice of tolerance range as in Fig. 5, the tolerance range of each link is shown in Table III.

Points	Px	Ру	Pz	d1 (mm)	a2 (mm)	d3+d4	d5+d6
<i>P</i> 1	-127.88	-302	408	650	700	675	300
1	-127.88	-301.8	408	650.2324	699.8633	674.6555	299.6622
2	-128.08	-302	408	650.1342	699.8544	674.9514	299.6714
3	-127.88	-302.2	408	649.7255	700.088	675.3291	300.3378
4	-127.68	-302	408	649.8237	700.0969	675.0331	300.3286
5	-127.88	-302	408.2	650.0981	699.8821	674.9626	300
6	-127.88	-302	407.8	649.8598	700.0692	675.0219	300
P2	-120	-288	410	650	700	675	300
1	-120	-287.8	410	650.3098	699.833	674.6188	299.5684
2	-120.2	-288	410	650.2908	699.7667	674.844	299.464
3	-120	-288.2	410	649.6515	700.1217	675.3646	300.4316
4	-119.8	-288	410	649.6705	700.1879	675.1394	300.5358
5	-120	-288	410.2	650.1024	699.8857	674.9581	300
6	-120	-288	409.8	649.8589	700.069	675.0253	300
<i>P</i> 3	200	300	250	650	700	675	300
1	200	300.2	250	650.0537	700.1361	674.8457	299.8346
2	199.8	300	250	650.2936	699.9129	674.7622	299.4519
3	200	299.8	250	649.9283	699.8518	675.1346	300.1654
4	200.2	300	250	649.6883	700.075	675.2181	300.5481
5	200	300	250.2	650.115	699.9434	674.9073	300
6	200	300	249.8	649.8669	700.0446	675.073	300
<i>P</i> 4	450	400	450	650	700	675	300
1	450	400.2	450	650.2455	700.2147	674.6973	299.3779
2	449.8	400	450	650.3459	699.9291	674.6647	299.3137
3	450	399.8	450	649.7254	699.7787	675.2713	300.6221
4	450.2	400	450	649.625	700.0642	675.3039	300.6863
5	450	400	450.2	650.0953	699.9761	674.8869	300
6	450	400	449.8	649.8757	700.0173	675.0817	300
P5	299	302	300	650	700	675	300
1	299	302.2	300	650.0724	700.1453	674.849	299.7822
2	298.8	302	300	650.2267	699.9132	674.8369	299.5793
3	299	301.8	300	649.919	699.8496	675.1413	300.2178
4	299.2	302	300	649.7648	700.0816	675.1534	300.4207
5	299	302	300.2	650.1205	699.9519	674.9096	300
6	299	302	299.8	649.8709	700.0429	675.0806	300

TABLE II: EXTRACTED RESULTS OF MEASURED TOLERANCES OF BUILT-UP LINKS IN ROBOT FANUC S900W

TABLE III: THE RESULTS OF DIMENSION TOLERANCES IN BUILT-UP LINKS OF ROPOT FANUC S000W

Links	d1 (mm)	a2 (mm)	d3+d4	d5+d6
Nominal dimensions	650	700	675	300
Upper limit tolerance	649.9283	699.9761	674.9626	299.8346
Lower limit tolerance	650.0537	700.0173	675.0219	300.1654
Tolerance range	0.1254	0.0412	0.0593	0.3308

Depending on the design requirement, these results can be used as the basis of joint clearance choice (Fig. 3) or allocation tolerances of link dimensions as illustrated in Fig.

## B. Parallel Robot

6.

Considering the tolerance calculation example of built-up links in 3RRR parallel robot with the deviation of the end-effector  $dR \le 0.01$  (mm) in all directions in Oxy coordinate (Fig. 9).

The structural nominal parameters of robot are: moving platform and base platform are equiangular triangles with side edges as h = 300 (mm), c = 600 (mm) respectively and the nominal dimensions of links  $a_i = 400$  (mm),  $b_i = 300$  (mm). The problem is determining the manufacturing tolerances of h, c,  $a_i$ ,  $b_i$  dimensions with the tolerance of the end-effector

≤0.01 (mm).



In Fig. 9, the points 1, 2, 3, 4 describe four allowable maximum deviation points of end-effector in the deviation cycle with center  $O_1$  and radius dR=0.01mm.

The center  $O_1$ , the desired nominal position of moving platform, and the given radius, dR=0.01mm, can be determined from coordinates of these points.

Considering the study points in the working space of robot, the inverse kinematic problem and applying the equations from (14) to (19), the study tolerances of the built-up links of 3RRR robot are presented in Table IV.

TABLE IV: EXTRACTED RESULTS OF MEASURED TOLERANCES IN BUILT-UP LINKS OF 3RRR PARALLEL ROBOT

Points	Px (mm)	Py (mm)	<i>a</i> (mm)	<i>b</i> (mm)	<i>c</i> (mm)	<i>h</i> (mm)
P1	181.205	403.802	400	300	600	300
1	181.195	403.802	400.0077	300.0057	599.9845	299.9787
2	181.215	403.802	399.9922	299.9942	600.0154	300.0212
3	181.205	403.792	399.9866	299.9899	599.992	300.0020
4	181.205	403.812	400.0133	300.0100	600.0079	299.9979
P2	213.081	419.724	400	300	600	300
1	213.071	419.724	400.0077	300.0058	599.9833	299.9775
2	213.091	419.724	399.9922	299.9941	600.0166	300.0224
3	213.081	419.714	399.9865	299.9898	599.9941	300.0042
4	213.081	419.734	400.0134	300.0101	600.0058	299.9957
P3	245.611	434.263	400	300	600	300
1	245.601	434.263	400.0078	300.0059	599.9821	299.9762
2	245.621	434.263	399.9921	299.9940	600.0178	300.0237
3	245.611	434.253	399.9863	299.9897	599.9962	300.0065
4	245.611	434.273	400.0136	300.0102	600.0037	299.9934
P4	278.794	447.237	400	300	600	300
1	278.784	447.237	400.0080	300.0060	599.9808	299.9748
2	278.804	447.237	399.9919	299.9939	600.0191	300.0251
3	278.794	447.227	399.9860	299.9895	599.9985	300.0089
4	278.794	447.247	400.0139	300.0104	600.0014	299.991
P5	312.647	458.344	400	300	600	300
1	312.637	458.344	400.0083	300.0062	599.9794	299.9732
2	312.657	458.344	399.9916	299.9937	600.0205	300.0267
3	312.647	458.334	399.9856	299.9892	600.0009	300.0117
4	312.647	458.354	400.0143	300.0107	599.999	299.9882
<i>P</i> 6	347.188	467.062	400	300	600	300
1	347.178	467.062	400.0086	300.0064	599.9779	299.9714
2	347.198	467.062	399.9913	299.9935	600.022	300.0285
3	347.188	467.052	399.9849	299.9887	600.0035	300.0147
4	347.188	467.072	400.01500	300.0112	599.9964	299.9852
P7	382.398	472.401	400	300	600	300
1	382.388	472.401	400.0091	300.0068	599.9762	299.9693
2	382.408	472.401	399.9908	299.9931	600.0237	300.0306
3	382.398	472.391	399.9841	299.9880	600.0065	300.0184
4	382.398	472.411	400.0158	300.0119	599.9934	299.9815

P8	417.965	472.138	400	300	600	300
1	417.955	472.138	400.0100	300.0075	599.974	299.9665
2	417.975	472.138	399.9899	299.9924	600.0259	300.0334
3	417.965	472.128	399.9826	299.9870	600.0102	300.0232
4	417.965	472.148	400.0173	300.0129	599.9897	299.9767
<i>P</i> 9	451.03	460.044	400	300	600	300
1	451.02	460.044	400.0115	300.0086	599.9712	299.9625
2	451.04	460.044	399.9884	299.9913	600.0287	300.0374
3	451.03	460.034	399.9799	299.9849	600.0151	300.0301
4	451.03	460.054	400.0200	300.0150	599.9848	299.9698
P10	464.521	428.604	400	300	600	300
1	464.511	428.604	400.0143	300.0107	599.9681	299.9573
2	464.531	428.604	399.9856	299.9892	600.0318	300.0426
3	464.521	428.594	399.9750	299.9812	600.0205	300.0392
4	464.521	428.614	400.0249	300.0187	599.9794	299.9607
P11	454.378	394.708	400	300	600	300
1	454.368	394.708	400.0174	300.0130	599.9665	299.9534
2	454.388	394.708	399.9825	299.9869	600.0334	300.0465
3	454.378	394.698	399.9697	299.9773	600.0233	300.0459
4	454.378	394.718	400.0302	300.0226	599.9766	299.9540
P12	435.335	364.645	400	300	600	300
1	435.325	364.645	400.0203	300.0152	599.9662	299.9509
2	435.345	364.645	399.9796	299.9847	600.0337	300.0490
3	435.335	364.635	399.9646	299.9735	600.0238	300.0503
4	435.335	364.655	400.0353	300.0264	599.9761	299.9496

Similarly, by applying the method of tolerance range determination as shown in Fig. 5, the tolerance range of the built-up links is presented in Table V.

TABLE V: THE TOLERANCE RESULTS OF BUILT-UP LINK DIMENTIONS OF
<b>3RRR</b> PARALLEL ROBOT

Links	<i>a</i> (mm)	<i>b</i> (mm)	<i>c</i> (mm)	<i>h</i> (mm)
Nominal dimensions	400	300	600	300
Upper limit tolerance	400.0077	300.0058	600.0009	300.0117
Lower limit tolerance	399.9922	299.9942	599.9991	299.9883
Tolerance range	0.01543	0.01157	0.00182	0.02341

Depending on the design requirement, these results can be used as the basis of joint clearance choice (Fig. 3) or allocation tolerances of link dimensions as illustrated in Fig. 6.

#### C. Checking the Accuracy of the Proposed Method

In robot design technique, the manufacturing tolerances of the build-up joints are calculated to meet the separated design requirements as accuracy, manufacture cost and so on. The measured tolerances must meet the requirement of the initial design.

Because the values of tolerances are small in nature, and the robot kinematic problem includes variety of transcendental functions which require round-off their numbers, the numerical method is an approximate approach. Hence,

Required tolerance = round-off error + method error.

The proposed method in this paper, however utilize the highly accurate GRG, the calculated results of tolerance are high accuracy and reliability. As a demonstration, an example is given as follows.

Taking the tolerance result of 3RRR parallel robot in which

the end-effector error dR  $\leq 0.01$ mm in all dimensions in working space.

Considering a trajectory across 12 key points belong to an ellipse in robot working space (Fig. 10). The tolerances result in Table V. The errors of the moving platform when following this trajectory are shown in Fig. 11.



Fig. 10. The moving trajectory across twelve points in an ellipse.



It can be seen that the errors of all the measured points on control trajectory are in the range of 0.005mm to 0.0067mm (smaller than 0.01mm). It means that the calculated tolerances of the component links satisfy the requirement on the accuracy of the initial design.

## VIII. CONCLUSION

The proposed method both considers the effect of link and joint tolerances to the drive system simultaneously and give the instruction of determining the tolerance of each link in the kinematic chain based on the nature of kinematic reaction of the movement accompanied with mathematical statistics. This method overcomes the drawback mathematic models as the squared requirement of inversed matrix in Jacobian matrix. In particularly, with the different mathematic models in different stage of mechatronics designation, the proposed method warrants the solidity of the object designed by applying a unique mathematic model to both kinematic and manufactures tolerance problems.

The solution of spatial dimensions with several constraints to allocate the tolerance of the end-effector into the tolerances of the component links is neglected. The results are calculated in both direct and reverse directions and experiment on mathematic model easily. Besides the widen application on different type of robot as serial robot, parallel robot which has either redundant or under actuated drive, the proposed method is more fitted with the nature of mechanism than the conventional method of establishing and solving the dimensional chain.

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