Analytical Based Inverse Kinematics Method for 5-axis Delta Robot

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Abstract—**Linear Delta Robot, or which is know as Parallel-Link Robot, is a three arms robot. The arms of robot was installed parallel with the central joint where the gripper was installed as the end-effector. Generally, the end-effector was aimed to pick and place an object in the workspace area of robot. The mechanism of gripper move at X, Y, Z axis was determined by calculating the inverse kinematics of every robot's arms. Then the result was used to convert the motion of gripper from linear motion to become rotation motion. In this study, the analytical method to calculate the invers kinematics of five-axis delta robot was developed. By using this method, the rotation of actuator to achieved the required position could be calculated. The proposed method was tested by program the end-effector to follow a complex trajectory. By using a simulation program that was developed in this study, the rotation of every motor could be calculated.**

Index Terms—**Parallell robot, delta robot, invers kinematics, analytical method.**

I. INTRODUCTION

In recent years, the parallel robot has been developed widely since the parallel manipulator exhibit high stiffness in nearly all configurations, high accuracy and high payload capacity. With these advantages, therefore, delta robot used widely as industrial robots, flight simulator, machine tools, etc. However, the major drawback of parallel robots is their relatively limited workspace. The comparison between serial robot and parallel robot was discussed in detail by Cox and Tesar [1].

Forward kinematics of robot deals with defining the end-effector position and orientation when the individual joint angles were given. Meanwhile the inverse kinematic problem is aimed to determine the required joint angle and position to achieve a specified end-effector position and orientation. Several methods to solve the inverse kinematics problem in various application have been proposed. Zhao and Norman [2] developed an inverse kinematics to define Cartesian space constraints by determining a local minimum of a set of non-linear formulation. The numerical method was developed to solve the inverse kinematics through a

series of calculation. The most popular numerical approach to fine a linear approximation to the inverse problem is Jacobian method. There are many approach have been proposed for calculating the Jacobian inverse, such as the Jacobian Transpose, Damped Least Squares (DLS), Damped Least Squares with Singular Value Decomposisiton (SVD-DLS), Selectively Damped Least Squares (SDLS) and several extensions [3]-[6].

Jacobian method produce accurate calculation, however, most of these approaches suffer from expensive computational cost, complex matrix calculations, and singularity problems [7]. It is due to the method solved the inverse kinematics fully numerically. To overcome the problem regarding high computational time, a hybrid analytical-numerical based methodology was proposed [8], [9]. A hybrid method proven shorter the computational time as compared to a fully numerical method. Several studies were showed that analytical method could decrease the computational time significantly [10]-[13].

Therefore, in this study, an analytical method was developed to calculate the inverse kinematics of five-axis parallel robot. In this study, the architecture of three-axis delta robot was improved by adding two additional degree of freedom at the end-effector. The end-effector can be rotated 360 degrees about Z-axis and 90 degrees about x-axis.

II. ANALYTICAL INVERSE KINEMATICS

The architecture of five-axis delta robot was designed as presented in Fig. 1(a). It is consist of a fixed platform, which is located at the base of robot, and a moving platform where the end-effector located. The moving platform was actuated by three units of stepper motor that was set-up at point a_1, a_2, and a_3. The stepper motor change the position of moving platform by moving the point a_1, a_2, and a_3 up dan down about *z-axis*. Two more actuators were located under the moving platform. These two actuators rotate the end-effector about z-axis and *y-axis*.

For the purpose of kinematical analysis of the moving end-effector, appropriate operator for the coordinate system transformations are required. Therefore, three coordinate frames, as illustrated in Fig. 1(a) and Fig. 1(b), were employed to represent the posistion and orientation of the end effector. These coordinate frames consist of: 1) the global coordinate system (GCS), which was set at the center of fixed platform as shown in Fig. 1(a), 2) the local coordinate system (LCS) is the original position of end-effector, which was located at point *E* as shown in Fig. 1(b), and 3) the moving coordinate system (MCS), which was also located at point *E* as shown in Fig. 1b. The GCS is a fixed frame system that is represented by the basis vector

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X, Y, Z, while LCS and MCS are denoted by x, y, z and u, v, y.

The z-axis of GCS is normal to the base and the *y-axis* is aligned with $b_1 b_2$ as shown in Fig. 1(a). The kinematics formulation was developed to define the motion of point $a_1(x_{a_1}, y_{a_1}, z_{a_1})$; $a_2(x_{a_2}, y_{a_2}, z_{a_2})$; $a_3(x_{a_3}, y_{a_3}, z_{a_3})$, the rotation about *z-axis* (ε), and the rotation about *y-axis* (λ). The location and orientation of end-effector was determined using the coordinate of three given points, point $D(x_D; y_D; z_D)$, E' $(x_{E'}; y_{E'}; z_{E'})$ and $F(x_{F'}; y_{F'}; z_{F'})$. Originaly, point E and F are aligned with *x-axis*. Based on these points, the coordinate of the the points could be determined and finally the rotataion of every motor could be calculated.
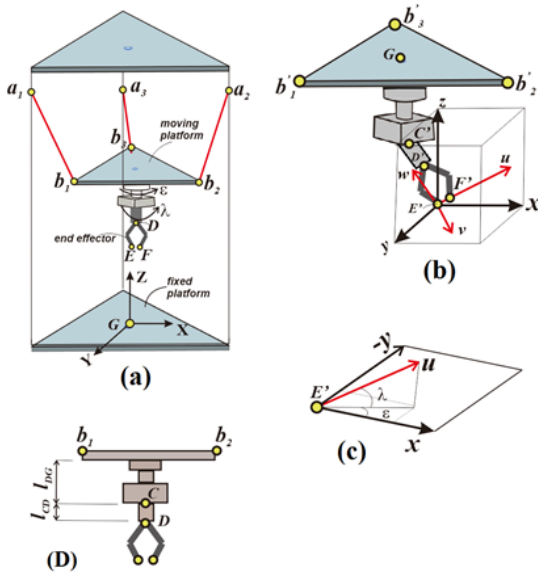

Fig. 1. (a) The architecture of robot delta, (b) moving platform and end-effector, (c) rotation angle of end-effector, (d) the distance between points.

The orientation of end-effector relatif to LCS is shown in Fig. 1(c) and it was calculated using the tool orientation reltive to the *x-axis* (ε) and the *z-axis* (λ).

$$\varepsilon = \sin^{-1}\left(\frac{y_{F'} - y_{E'}}{x_{F'} - x_{E'}}\right) \tag{1}$$

$$\lambda = \sin^{-1}\left(\frac{z_{F'} - z_{E'}}{\sqrt{(x_{F'} - x_{E'})^2 + (y_{F'} - y_{E'})^2}}\right) \tag{2}$$

The rotation of ε and λ were used to acutate Motor 1 and Motor 2, subsequently. The operator $[T]$ was used to transform the coordinate frame from the MVS to LCS, which involved the end-effector rotation about the y-axis and the z-axis. It was calculated as follows,

$$[T] = Rot(Z, \varepsilon). Rot(Y, \lambda) \tag{3}$$

$$[T] = \begin{bmatrix} \cos\varepsilon & -\sin\varepsilon & 0 \\ \sin\varepsilon & \cos\varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\lambda & 0 & \sin\lambda \\ 0 & 1 & 0 \\ \sin\lambda & 0 & \cos\lambda \end{bmatrix} \tag{4}$$

By referring Then, point C', whics is located at the pin of rotation about *x-axis*, was defined as,

$$C'(x_{C'}; y_{C'}; z_{C'}) = [T] \, E' \, x \, D(x_D; y_D; z_D) + (0; 0; l_{CD}) \tag{5}$$

where l_{CD} is the distance between point C and D as shown in Fig. 1(d).

By referring to Fig. 1(a), the coordinate of $G'(x_{G'}; y_{G'}; z_{G'})$, which is located at the center of moving platform, could be calculated as follow,

$$\begin{aligned} G'(x_{G'}; y_{G'}; z_{G'}) \\ = C'(x_{C'}; y_{C'}; z_{C'}) \\ + (0; 0; +l_{DG}) \end{aligned} \tag{6}$$

where l_{DG} is the distance between point D and G as shown in Fig. 1d. Then all connecting point on the moving platform could be determined as follow,

$$b'_1(x_{b'_1}; y_{b'_1}; z_{b'_1}) = G'(x_{G'}; y_{G'}; z_{G'}) + (-r. \sin\alpha; \; r. \cos\alpha; \; 0) \tag{7}$$

$$b'_2(x_{b'_2}; y_{b'_2}; z_{b'_2}) = G'(x_{G'}; y_{G'}; z_{G'}) + (0; -r; 0) \tag{8}$$

$$\begin{aligned} b'_3(x_{b'_3}; y_{b'_3}; z_{b'_3}) = \\ G'(x_{G'}; y_{G'}; z_{G'}) + (r. \sin\alpha; \; r. \cos\alpha; \; 0) \end{aligned} \tag{9}$$
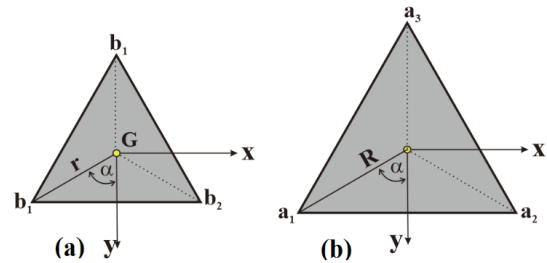

Fig. 2. Moving platform and upper platform.

where r is the distance between connecting points (b'_1, b'_2, b'_3) to the point G. In this case, the triangle designed fo moving platform is an equilateral triangle as shown in Fig. 2(a). Point b'_1, b'_2, b'_3 are located on the moving platform, and hence, they can move freely in *x-axis*, *y-axis*, and *z-axis*. However for point a'_1, a'_2, a'_3, they are installed at the column, as shown in Fig. 1(a), and hence they can only move in Z-axis. Therefore, the coordinate of a'_1, a'_2, a'_3 in X-axis and Y-axis were defined as,

$$\begin{aligned} a'_1(x_{a'_1}; y_{a'_1}) = b'_1(x_{b'_1}; y_{b'_1}) + ((R - \\ r). \sin\alpha; \; (R - r). \cos\alpha) \end{aligned} \tag{10}$$

$$a'_2(x_{a'_2}; y_{a'_2}) = b_2(x_{b'_2}; y_{b'_2}) + (0; (r - R)) \tag{11}$$

$$\begin{aligned} a'_3(x_{a'_3}; y_{a'_3}) = b_3(x_{b'_3}; y_{b'_3}) + ((R - \\ r). \sin\alpha; \; (R - r). \cos\alpha) \end{aligned} \tag{12}$$

Finally, the Z-axis of a'_1, a'_2, a'_3 were calculated as follow,

$$z_{a'_n} = \pm\sqrt{L^2 - (x_{b'_n} - x_{a'_n})^2 - (y_{b'_n} - y_{a'_n})^2} \tag{13}$$

where n is 1, 2, 3 and L is the length of link that connecting point a and point b. The displacement of point a'_1, a'_2, a'_3 was actuated by Motor 3, Motor 4, and Motor 5. The rotation of motor required to achieved the expected position (ϑ) was calculated as follow,

$$\vartheta_n = \frac{z_{a'_n} - z_{a_n}}{2.\pi.r_s} \times 360 \qquad (14)$$

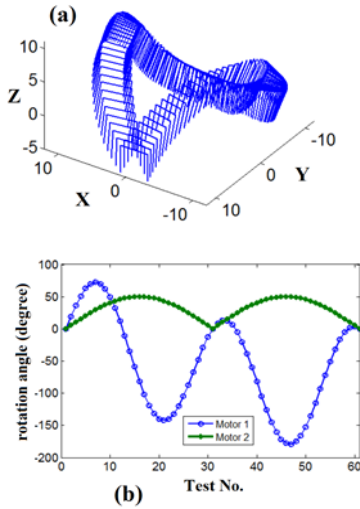where r_s is the radius of slider wheel to move point a'_1, a'_2, a'_3 up and down.

Fig. 3. (a) end-effector trajectory, (b) the rotation angle of Motor 1 and Motor 2.

III. IMPLEMENTATION AND DISCUSSION

All the equation derived in the previous section have been used to develop a simulation program using Matlab. The program is called Analytical Inverse Kinematics Simulation. The proposed method was tested to generate all variable required to actuate the end-effector to the designed position. The variables required by five motor used are $\varepsilon, \lambda, \vartheta_1, \vartheta_2, \vartheta_3$.

To ensure the implementation of the proposed method, one test was performed. In this test, the end-effector was set to follow a complex trajectory as shown in Fig. 3(a). During following the trajectory, the orientation and position of the end-effector was continuously change. Using the proposed method, the rotation of the Motor 1 about *z-axis* and the rotation of Motor 2 about *x-axis* could be calculated and the result were presented in Fig. 3(b). From this figure can be seen that the rotation angle of the Motor 1 and Motor 2 were very fluctuating. The fluction of rotation angle was happened because of the designed trajectory was very complex.

Using the developed program simulation, the rotation angle of Motor 3, Motor 4, and Motor 5, could also be calculated and the result were presented in Fig. 4(a). The rotation angle was calculated based on the position of upper link in *z-axis*. The position of upper link during the motion of robot was presented in Fig. 4(b). From Fig. 4 can be seen that the rotation angle of the motor are very dynamic. It was occurred due to the complexity of trajectory.

Eventhough the method showed that it can be used to define the inverse kinematics of 5-axis delta robot, the accuracy of the model need to be validated. In this study, the verification was performed using two mechanisms. First, the accuracy was tested by comparing the coordinate of point a'_1, a'_2, a'_3 using commercial CAD software Siemens-NX. In this test, there are 60 data were taken and the results are presented in Fig. 5. From this figure can be

seen that all the points obtained from Siemens-NX coincided with the trajectory generated using the program simulation. This demonstrate that the proposed method was accurate.
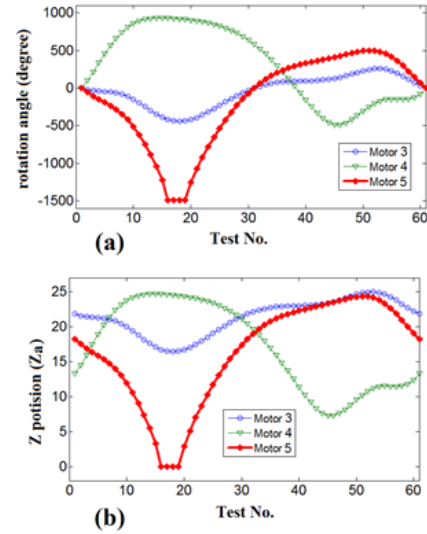

Fig. 4. (a) The rotation angle of Motor 3, Motor 4, Motor 5, and (b) the z-axis of upper platform.
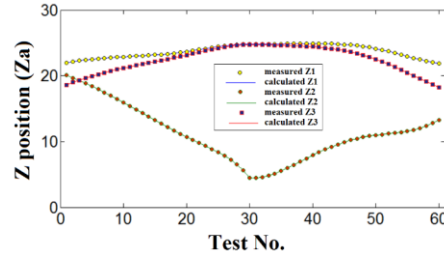

Fig. 5. Model verification using Siemens-NX.

The second verification was performed using experimental test. In this study, the delta robot was also constructed as presented in Fig. 6a-Fig.6c. Using this robot, the second verification was performed. The verification was performed to check the ability of the physical robot in following the trajectory. The result was presented in Fig. 6d. From this figure can be seen that the parallel robot can follow the designed trajectory well. The deviation were relatively small.
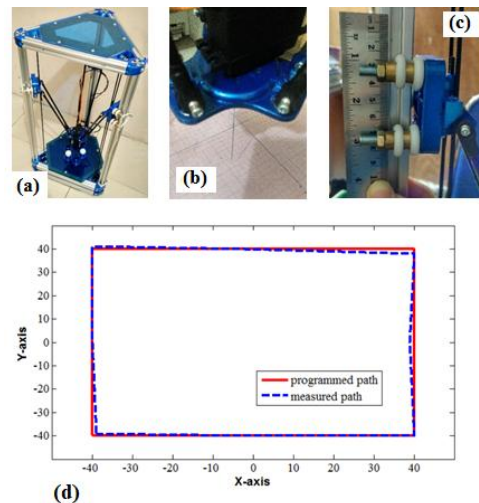

Fig. 6. a) construction of parallel robot, b) the method to verify the position of end-effector in x-axis and y-axis, c) the method to verify the position of mobile platform in z-axis, d) the verification result on the accuracy of robot in following the programmed trajectory.

IV. Conclusion

In this study, an analytical method to calculate the inverse kinematics of delta robot has been developed. The method was derived for a five-axis delta robot. All the equation derived were used to develop a simulation program. One test to verify the ability of the proposed method in calculating the rotation of each motor to achieved the desired position was performed. The result show that the method could generate the data well. The main advantage of analytical method comapared to numerical based methodology is computationally efficient.

Acknowledgment

References

[1] D. J. Cox and D. Tesar, "The dynamic modeling and command signal formulation for parallel multi-parameter robotic devices," Internal Report CIMAR, University of Florida, Gainesville, FL, September 1981.
[2] J. Zhao and N. I. Badler, "Inverse kinematics positioning using nonlinear programming for highly articulated figures," *ACM Transactions on Graphics (TOG)*, vol. 13, no. 4, 1994, pp. 313-336.
[3] V. Matikainen, J. Backman, and A. Visala, "Cartesian control of an advanced tractors rear hitch–damped least-squares solution," *World Congress*, vol. 19, no. 1, 2014.
[4] S. R. Buss and J. S. Kim, "Selectively damped least squares for inverse kinematics," *Journal of Graphics, GPU, and Game Tools*, vol. 10, no. 3, pp. 37-49, 2005.
[5] I. Dulęba and M. Opałka, "A comparison of Jacobian-based methods of inverse kinematics for serial robot manipulators," *International Journal of Applied Mathematics and Computer Science*, vol. 23, no. 2, pp. 373-382, 2013.
[6] B. Hu, "Formulation of Unified Jacobian for Serial-Parallel Manipulators," *Robotics and Computer-Integrated Manufacturing*, vol. 30, no. 5, pp. 460-467, 2014.
[7] A. Aristidou and J. Lasenby, "Inverse kinematics: A review of existing techniques and introduction of a new fast iterative solver," University of Cambridge, Department of Engineering, 2009.
[8] A. L. Olsen and H.G. Petersen, "Inverse kinematics by numerical and analytical cyclic coordinate descent," *Robotica*, vol. 29 no. 4, pp. 619-626, 2011.
[9] B. Kenwright, "Inverse kinematics–cyclic coordinate descent (CCD)," *Journal of Graphics Tools*, vol. 16, no. 4, pp. 177-217, 2012.
[10] G. Kiswanto, H. Hendriko, and E. Duc, "An analytical method for obtaining cutter workpiece engagement during a semi-finish in five axis milling," *Computer-Aided Design*, vol. 55, pp. 81-93, 2014.
[11] G. Kiswanto, H. Hendriko, and E. Duc, "A hybrid analytical and discrete based methodology for determining cutter workpiece engagement in five-axis milling," *The International Journal of Advanced Manufacturing Technology*, vol. 80 no. 9-12, pp. 2083-2096, 2015.
[12] H. Hendriko, "Mathematical model for chip geometry calculation in five-axis milling," *Jurnal Teknologi*, vol. 77, no. 23, pp. 108-113, 2015.
[13] G. Kiswanto, H. Hendriko, and E. Duc, "Analytical method for obtaining cut geometry of helical toroidal cutter during semi-finish in 5-Axis milling," *Applied Mechanics and Materials*, Trans Tech Publications, vol. 541, pp. 780-784, 2014.

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