Natural Frequencies of Conical Boring Bars

Yao Donghui and Ren Yongsheng

***Abstract*—In the process of boring, the proper selection for the section shape of a boring bar can improve the quality and efficiency of mechanical processing by suppressing its vibration. The boring bar can be simplified as a cantilever beam with variable cross-section. By employing a shape function of the cross-section, the first and second natural frequencies of cantilever beams with different cross-section shapes are investigated via assumed modes method. The influence of the taper and the convexity of the cross section on natural frequency is also analyzed. By solving the principal stiffness and mass of each mode, the influence of the cross-section shape on principal stiffness and mass is studied. By numerical calculating, the influence of the change rate of the corresponding principal stiffness and mass on natural frequency is analyzed. Finally, the design principles of variable cross-section cantilever beam are given.**

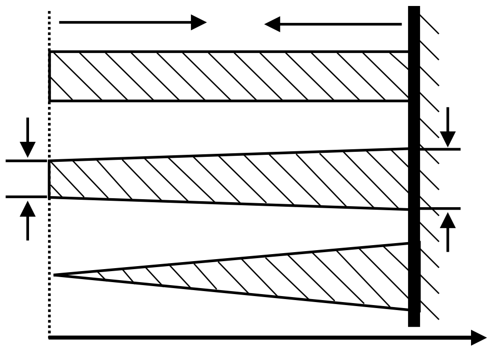
***Index Terms*—Conical boring bar, variable cross-section beam, assumed modes method, natural frequency, principal stiffness.**

I. Introduction

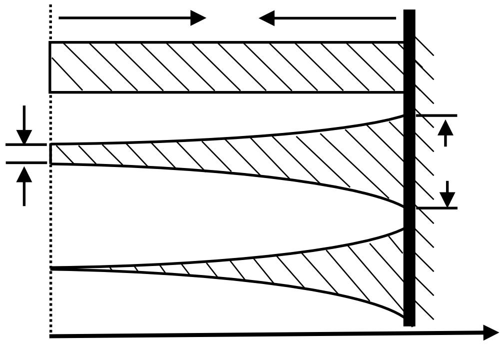
Boring bar is widely used in mechanical processing. Longer boring bars are required to process deeper holes. The slenderness makes the boring bar more likely to flutter under the exciting force. By designing the shape of boring bar's cross-section, its natural frequency can be changed, vibration can be suppressed, stability can be enhanced, and processing quality and accuracy can be improved.

Conical boring bars can be simplified as cantilever beams with variable cross-section, the vibration of which has been extensively studied. The calculation of the natural frequencies and vibration modes for a cantilever beam with linearly variable cross section can be done via exact or approximate methods [1]. The vibration characteristics of variable cross-section beams are studied in literature [3]-[6]. In literature [7]-[10], the vibration characteristics of conical beams are studied. The transverse vibration of wedge beam is studied in literature [11], [12]. The Assumed modes method is used to solve natural frequencies and modes of beams in literature [13]-[15]. Exact solutions of variable cross-section beams are determined in terms of Bessel's functions in literature [16]. Previous studies have not analyzed the combined effect of taper and convexity on frequency, and have not explained the change of frequency with section shape.

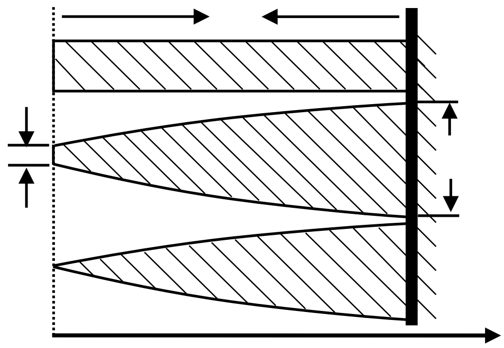
In the present study, the assumed modes method is used to solve the transverse vibration characteristics of conical beams, while a cross-section shape function is established to describe different shapes. The effects of taper and convexity of different shapes on the vibration characteristics are studied. The influence of cross-section shape on the first and second natural frequencies is given, and the trend of frequency variation is explained by the change rate of the principal stiffness and mass of each order.



(a)



(b)



(c)

Fig. 1. Schematic diagram of variable cross-section cantilever beam: (a) k=0, (b) k<0, (c) k>0.

II. Model of the Beam

In the present study, the conical boring bar is simplified as

Manuscript received January 17, 2019; revised May 22, 2019. This work was supported by the National Natural Science Foundation of China (No. 11672166 and No. 11272190).

The authors are with College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, Qingdao 266590, China (e-mail: 27110292@qq.com, renys@sdust.edu.cn).

doi: 10.18178/ijmmm.2019.7.6.472

a cantilever beam with variable cross section. The model is shown in Fig. 1. The heights of the free and clamped ends are a and b respectively. Take the free end as the origin of coordinate *x*. The width of the beam is *w*.

In order to study the influence of the section's taper and convexity on natural frequencies of variable cross-section beams, the nondimensional shape function of longitudinal section along the x-axis is introduced as follows:

$$S(x)=\frac{a}{b}+\left(1-\frac{a}{b}\right)\times\left(\frac{x}{L}\right)-k\left\lfloor\left(\frac{x}{L}\right)^2-\frac{x}{L}\right\rfloor \quad (1)$$

where L is the length of the beam; The ratio a/b designates the sharpness of the free end. $a/b=1$ corresponds to the equal cross-section beam. k is the convexity coefficient of the cross-section. As shown in Figure 1, $k=0$ corresponds to the wedge cross-section, $k<0$ corresponds to the concave cross-section, $k>0$ corresponds to the convex cross-section; The shape function is composed of a trapezoid function and a parabola function. The vertex of the parabola function is at $x=L/2$, which satisfies $S(0)=a/b$ and $S(L)=1$.

By substituting different a/b and k into shape function $S(x)$, different shapes with constant height of clamped end can be obtained. In order to study the influence of shape on the natural frequency of a beam when the total mass of the beam remains unchanged, the shape function $S(x)$ needs to be adjusted. For homogeneous materials, in order to make beams of different shapes equal in mass, beams of different shapes need to be equal in volume. The volume of the beam is:

$$V=\int_0^L S(x)w dx \quad (2)$$

where the section width w is a constant that can be put out of the integral symbol.

The adjusted shape function with constant total mass is defined as:

$$\bar{S}(x)=\frac{LS(x)}{\int_0^L S(x)dx} \quad (3)$$

Assuming that the moment of inertia of the clamped end is I_0 and the area of its cross section is A_0, we obtain:

$$\begin{aligned} \rho A(x) &= \rho A_0 L\times S(x) \\ EI(x) &= EI_0 S(x)^3 \end{aligned} \quad (4)$$

III. Assumed Modes Method

Then the natural frequencies of beams with variable cross-section are derived by assumed modal method.

Assume that the modes of the system are

$$\varphi_i(x)=(1-\frac{x}{L})^2(\frac{x}{L})^{i-1}, i=1,2,... \quad (5)$$

It is not difficult to verify that they satisfy boundary conditions of both displacement and force. To investigate the first two modes, three modes are assumed as:

$$\begin{aligned} \varphi_1(x) &= (1-\frac{x}{L})^2 \\ \varphi_2(x) &= (1-\frac{x}{L})^2(\frac{x}{L}) \\ \varphi_3(x) &= (1-\frac{x}{L})^2(\frac{x}{L})^2 \end{aligned} \quad (6)$$

Mass matrix M and stiffness matrix K:

$$\begin{aligned} M &= [m_{ij}] \\ K &= [k_{ij}] \end{aligned} \quad (7)$$

where

$$\begin{aligned} m_{ij} &= \int_0^L \rho A(x)\varphi_i\varphi_j dx \\ k_{ij} &= \int_0^L EI(x)\varphi_i''\varphi_j'' dx \end{aligned} \quad (8)$$

Substitute Eq. 4 into Eq. 8 can obtain:

$$\begin{aligned} m_{ij} &= \rho A_0 L\int_0^L S(x)\varphi_i\varphi_j dx \\ k_{ij} &= EI_0\int_0^L S(x)^3\varphi_i''\varphi_j'' dx \end{aligned} \quad (9)$$

The eigen equation is:

$$\lfloor K-M\omega^2 \rfloor = 0 \quad (10)$$

The frequency matrix ω and the corresponding mode matrix Φ can be obtained by solving Eq. 10. By extracting the constant part of Eq. 10, the nondimensional form of frequency is obtained:

$$\omega=\bar{\omega}\sqrt{\frac{EI_0}{\rho A_0 L^4}} \quad (11)$$

where $\bar{\omega}$ is the nondimensional frequency.

The principal mass and stiffness of each order can be obtained by the flowing equation:

$$\begin{aligned} M_p &= \Phi^T M\Phi \\ K_p &= \Phi^T K\Phi \end{aligned} \quad (12)$$

To study the influence of the change rate of the principal mass and stiffness with respect to taper and convexity, the change rate is defined as:

$$v_{i-1}=\frac{x_i-x_{i-1}}{|x_{i-1}|}, i=2,3,... \quad (13)$$

The principal stiffness, mass and frequency can be reduced to similar order of magnitude by Eq. 13.

IV. Numerical Results

A. The Influence of Taper

Firstly, the natural frequency, principal stiffness and principal mass of beams in different shapes with constant height of the clamped end are studied.

Taking *a*/*b* from 0 to 1, and *k*=0, keeping the clamped end height unchanged, results are calculated and plotted in Fig. 2 to Fig. 4. The calculation results are substituted into Eq. 13 to obtain the change rates of natural frequency, principal stiffness, and principal mass with *a*/*b*. The principal mass and stiffness varying with *a*/*b* are plotted in Fig. 2, in which Fig. 2(a) is for principal mass and Fig. 2(b) is for principal stiffness. As we can see from Fig. 2, the second-mode principal mass is smaller than the first-mode principal mass, and the second principal stiffness is larger than the first principal stiffness. With the decrease of taper (*a*/*b* increases), the principal mass and stiffness of each mode increase. The growth rates of the principal mass and stiffness with *a*/*b* are plotted in Fig. 3. It can be seen that the growth rate of the first principal stiffness is lower than that of the first principal mass, so the frequency decreases with the increase of *a*/*b*. In other words, the higher the taper (*a*/*b* decreases), the higher the first-mode frequency. It can be seen from Fig. 4(a) that the second-mode frequency first decreases and then begins to increase from the point near *x*=0.1. By comparing the growth curve of Fig. 4(b) with the second-mode frequency curve on Fig. 4(a), we can see that when the second principal stiffness growth curve intersects the second principal mass growth curve, the frequency reaches the minimum.

Next, the relationship between the principal vibration and taper of the cantilever beam with constant total mass is studied.

In order to keep the total mass of the beam constant, the calculation is carried out after replacing the shape function Eq. 1 with Eq. 3. As we can see from Fig. 5 and Fig. 6, the characteristics of the first-mode frequency, mass, and stiffness are similar with the case of beam with constant height of clamped end, except for that the inflection point moves to the right slightly. It can be seen from Fig. 7 that the second-mode frequency decreases with the increase of *a*/*b*, which is contrary to the case of beam with constant height of clamped end.

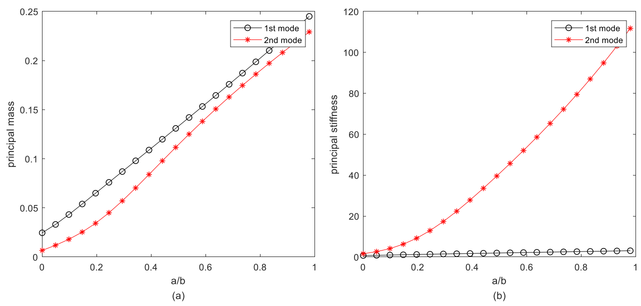


Fig. 2. The principal mass and stiffness varying with *a*/*b* (*k*=0, constant height of clamped end).

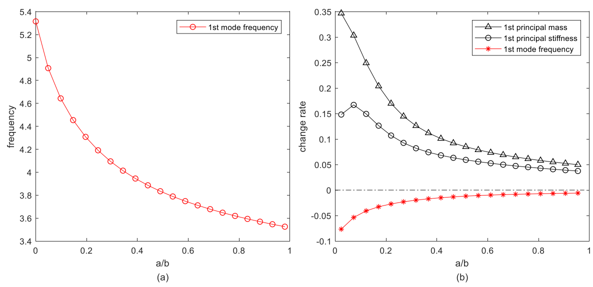


Fig. 3. The first mode natural frequency corresponds to the rate of change of the first principal mass and stiffness with *a*/*b* (*k*=0, constant height of clamped end).

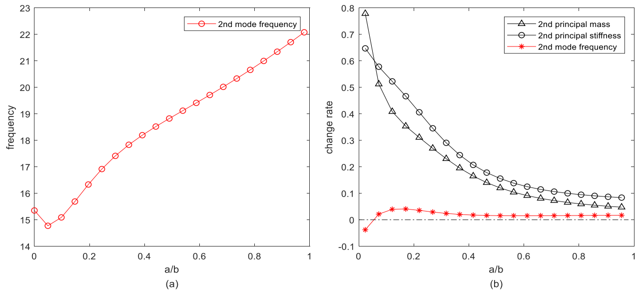


Fig. 4. Second-mode natural frequencies and the rate of change of second principal stiffness, mass and frequency (*k*=0, constant height of clamped end).

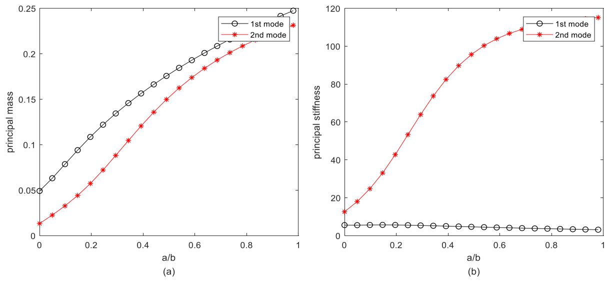


Fig. 5/ The principal mass and stiffness of each mode varying with *a*/*b* (*k*=0, constant mass).

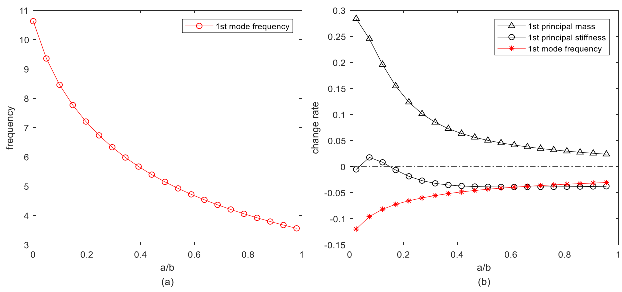


Fig. 6. The first mode natural frequency corresponds to the change rates of the first principal mass and stiffness with *a*/*b* (*k* = 0, constant mass).

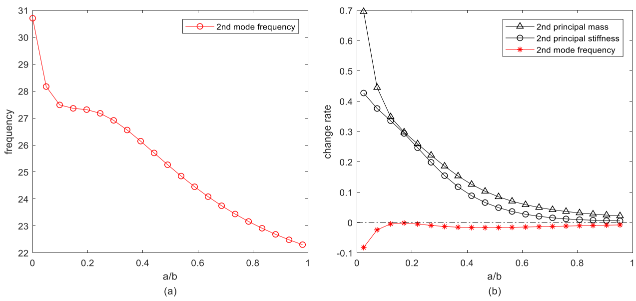


Fig. 7. The second mode natural frequency and the change rate of second principal stiffness, mass and frequency (*k* = 0, constant mass).

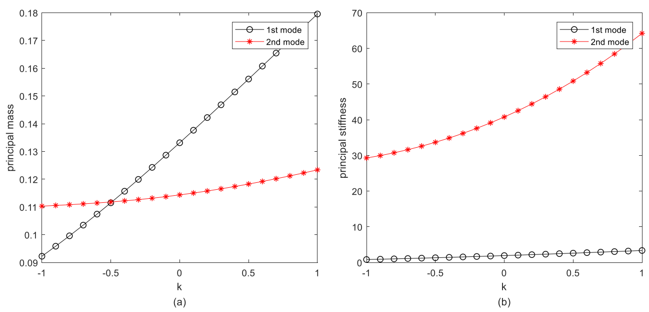


Fig. 8. The principal mass and stiffness varying with *k* (*a*/*b* = 0.5, with constant height of clamped end).

B. The Influence of Convexity

Firstly, the influence of the convexity on frequencies of beams with constant height of the clamped end are studied.

Taking a/b=0.5 and k from -1 to +1, keeping the clamped end height unchanged, the principal mass, principal stiffness, frequency and their change rate are calculated. The results are plotted in Fig. 8 to Fig. 10. It can be seen from Eq. 8 to Eq. 10 that the first-mode and second-mode frequencies increase with the increase of k. The principal stiffness increases faster than the principal mass. It means convex beams have higher first-mode natural frequencies. It is easy to verify that the average diameter and total mass of the beam increase with the increase of k.

Next, the influence of the convexity on frequencies of beams with constant mass is studied.

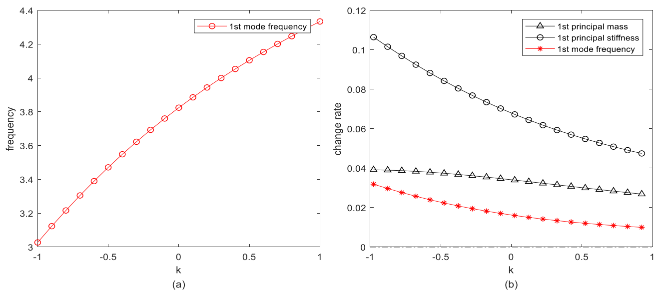


Fig. 9. The first mode natural frequency corresponds to the change rate of the first principal stiffness, mass and frequency (a/b = 0.5, constant height of clamped end).

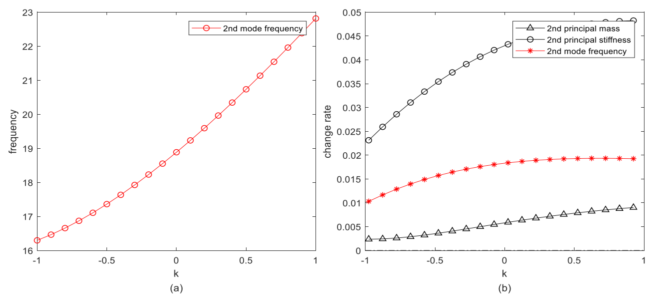


Fig. 10. Second-mode natural frequencies and the change rate of the second principal stiffness, mass and frequency (a/b = 0.5, constant height of clamped end).

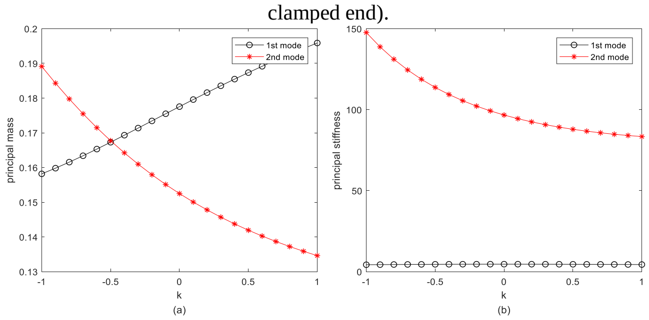


Fig. 11. Curves of principal mass and principal stiffness varying with k (a/b=0.5, constant mass).

It can be seen from Fig. 11 that the first-mode principal stiffness and mass increase with the increase of k. The principal stiffness increases slower than the principal mass, so the first-mode frequency decreases with the increase of k in Fig. 12(a). Fig. 11 also shows that the second principal stiffness and mass decrease with the increase of k. The second-mode principal stiffness decreases faster than the second-mode principal mass, so the second-mode frequency decreases with the increase of k in Fig. 13(a). It can be seen from Fig. 12(b) that the stiffness curve and the mass curve of the first mode intersect at k=0.3, which corresponds to the inflection point of the frequency curve in Fig. 12(a). Fig. 13(a) shows that the first-mode frequency first increases and then decreases. It can be seen from Fig. 13(b) that the stiffness curve and the mass curve of the second mode intersect at k=0.7, which corresponds to the inflection point of the frequency curve in Fig. 13(a). On the whole, the change of frequency with k of beams with constant mass is contrary to beams with constant height of clamped end. The reason the principal stiffness of beams with constant height of clamped end increases faster with k than that of principal mass. The thinner the beam with constant mass is, the faster the stiffness decreases.

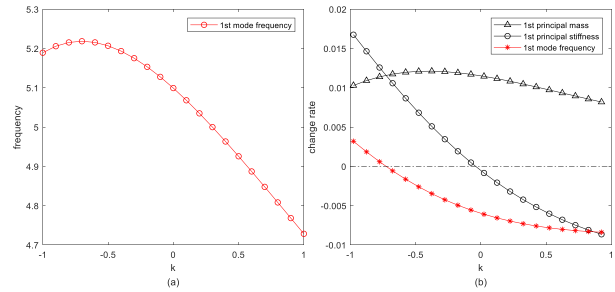


Fig. 12. The first mode natural frequency and the change rate of first principal stiffness, mass and frequency (a/b=0.5, constant mass).

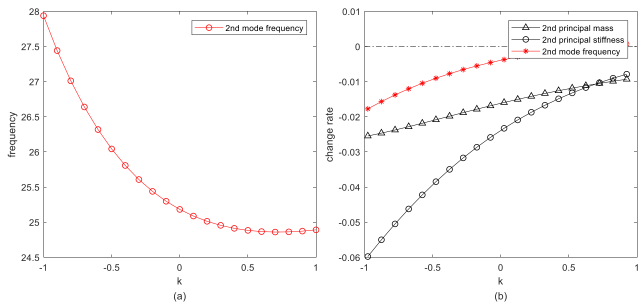


Fig. 13. The second mode natural frequency and the change rate of second principal stiffness, mass and frequency (a/b=0.5, constant mass)

TABLE I: THE MATERIAL PROPERTIES

E(GPa)	v	L(m)	b(m)	w(m)	ρ(kg/m^3)
200	0.3	20	1	1	7800

TABLE II: COMPARATION OF DIFFERENT METHODS

a/b	Assumed modes		3D-Elastic		Diff. %	
	1st mode	2nd mode	1st mode	2nd mode	1st mode	2nd mode
0	3.0914	8.9289	3.0867	8.8125	0.15	1.32
0.5	2.2300	10.949	2.2204	10.568	0.43	3.61
1	2.0504	12.841	2.0409	12.643	0.07	1.57

C. Comparation with 3D-elastic model via FEM

To verify the modal system, the 3D-Elastic model via FEM is used to calculate some typical shapes of the beam. Because the results above are in nondimensional form, material properties in Table I are substituted into Eq. 11 and divided by 2π to accord with the results from the 3D-Elastic model. Taking k=0 and a/b=[0,0.5,1], the results from the two methods are compared in Table II. It can be seen from Table 2 that the predicted frequencies are slightly higher. The mode functions assumed in Eq. 6 are suitable for the present study.

V. CONCLUSION

In the present study, a shape function of the beam's cross-section has been used to investigate the relationship between the vibration characters and the shape. The frequency, principal stiffness, principal mass and their change rates under the condition of keeping the height or mass unchanged have been studied. The following conclusions are drawn:

1) The difference between the change rate of principal stiffness and mass leads to different trends of natural frequencies. When the principal stiffness increases faster than principal mass, the natural frequency rises and vice versa.

2) Selection of taper (a/b)

Taper beams have higher first-mode frequencies and lower second-mode frequencies than equal-section beams. If possible, shapes with lower a/b (more taper) should be adopted to obtain higher first-mode natural frequencies.

3) Selection of convexity (k)

When the height of clamped end is constant, shapes with $k > 0$ (more convex) should be adopted to obtain higher first-mode natural frequencies, which is suitable for the situation where the size of clamped end is limited. When the mass is constant, $k < 0$ (more concave) should be adopted, which is suitable for the situations sensitive to weight or material consumption.

REFERENCES

[1] M. Attar, "A transfer matrix method for free vibration analysis and crack identification of stepped beams with multiple edge cracks and different boundary conditions," *International Journal of Mechanical Sciences*, vol. 57, no. 1, pp. 19-3, 2012.

[2] G. D. Anderson, M. K. Vamanamurthy, and M. Vuorinen, *Conformal Invariants, Quasiconformal Maps, and Special Functions, Quasiconformal Space Mappings*, Springer Berlin Heidelberg, 1992.

[3] C. Cui, "A solution for vibration characteristic of Timoshenko beam with variable cross-section," *Journal of Dynamics & Control*, vol. 10, no. 3, pp. 258-262, 2012.

[4] M. C. Ece, M. Aydogdu, and V. Taskin, "Vibration of a variable cross-section beam," *Mechanics Research Communications*, vol. 34, no. 1, pp. 78-84, 2007.

[5] J. Jaroszewicz and L. Zoryi, "Investigation of the effect of axial loads on the transverse vibrations of a vertical cantilever with variable parameters," *International Applied Mechanics*, vol. 36, no. 9, pp. 1242-1251, 2000.

[6] D. I. Caruntu,"Dynamic modal characteristics of transverse vibrations of cantilevers of parabolic thickness," *Mechanics Research Communications*, vol. 36, no. 3, pp. 391-404, 2009.

[7] D. Zhou and Y. K. Cheung, "Vibrations of tapered timoshenko beams in terms of static timoshenko beam functions," *Journal of Applied Mechanics*, vol. 68, no. 4, pp. 596-602, 2001.

[8] N. M. Auciello and G. Nolè, "Vibrations of a cantilever tapered beam with varying section properties and carrying a mass at the free end," *Journal of Sound & Vibration*, vol. 214, pp. 105-119, 1998.

[9] R. P. Goel, "Transverse vibrations of tapered beams," *Journal of Sound & Vibration*, vol. 47, no. 1, pp. 1-7, 1976.

[10] G. Genta and A. Gugliotta, "A conical element for finite element rotor dynamics," *Journal of Sound & Vibration*, vol. 120, no. 1, pp. 175-182, 1988.

[11] S. Naguleswaran, "A direct solution for the transverse vibration of euler-bernoulli wedge and cone beams," *Journal of Sound & Vibration*, vol. 172, no. 3, pp. 289-304, 1994.

[12] H. D. Conway and J. F. Dubil, "Vibration frequencies of truncated-cone and wedge beams," *Journal of Applied Mechanics*, vol. 32, pp. 932-935, April 1965.

[13] S. Timoshenko, D. H. Young, and W. Weaver, *Vibration Problems in Engineering*, 4th ed., Wiley, New York, 1974.

[14] S. O. R. Moheimani and R. L. Clark, "Minimizing the truncation error in assumed modes models of structures," *Journal of Vibration and Acoustics*, vol. 4, pp. 2398-2402, February 2000.

[15] P. D. Cha and W. C. Wong, "A novel approach to determine the frequency equations of combined dynamical systems," *Journal of Sound & Vibration*, vol. 219, pp. 689–706, July 1999.

[16] S. Timoshenko and S. P. Stephen, *History of Strength of Materials: With a Brief Account of the History of Theory of Elasticity and Theory of Structures*,McGraw-Hill Book Company, Inc. New York, 1983.

Yao Donghui was born on December 15, 1981 in Chengde, Hebei Province, China. Currently, he is a PhD student at the College of Mechanical and Electronic Engineering, Shandong University of Science and Technology. He is studying about composite material mechanics and mechanical vibration.

Ren Yongsheng was born on July 9, 1956 in Taiyuan, Shanxi Province, China. He is a professor at the College of Mechanical and Electronic Engineering, Shandong University of Science and Technology. His main research fields are mechanical system dynamics, non-linear vibration, composite material mechanics, intelligent material structures, and vibration control.